

A hybrid computational intelligence model for foreign exchange rate forecasting

Mehdi Khashei^{1*}; Farimah Mokhatab Rafiei²; Mehdi Bijari³; Seyed Reza Hejazi⁴

¹ Ph.D. Student, Dept. of Industrial Engineering, Isfahan University of Technology, Isfahan, Iran

² Assistant Professor, Dept. of Industrial Engineering, Isfahan University of Technology, Isfahan, Iran

^{3,4} Associate Professor, Dept. of Industrial Engineering, Isfahan University of Technology, Isfahan, Iran

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Abstract: Computational intelligence approaches have gradually established themselves as a popular tool for forecasting the complicated financial markets. Forecasting accuracy is one of the most important features of forecasting models; hence, never has research directed at improving upon the effectiveness of time series models stopped. Nowadays, despite the numerous time series forecasting models proposed in several past decades, it is widely recognized that exchange rates are extremely difficult to forecast. Artificial Neural Networks (ANNs) are one of the most accurate and widely used forecasting models that have been successfully applied for exchange rate forecasting. In this paper, a hybrid model is proposed based on the basic concepts of artificial neural networks in order to yield more accurate results than the traditional ANNs in short span of time situations. Three exchange rate data sets—the British pound, the United States dollar, and the Euro against the Iran rial—are used in order to demonstrate the appropriateness and effectiveness of the proposed model. Empirical results of exchange rate forecasting indicate that hybrid model is generally better than artificial neural networks and other models presented for exchange rate forecasting, in cases where inadequate historical data are available. Therefore, our proposed model can be a suitable alternative model for financial markets to achieve greater forecasting accuracy, especially in incomplete data situations.

Keywords: Computational intelligence; Artificial Neural Networks (ANNs); Fuzzy logic; Time series forecasting; Financial markets; Exchange rate

1. Introduction

Foreign exchange markets are among the most important and the largest financial markets in the world with trading taking place twenty-four hours a day around the globe and trillions of dollars of different currencies transacted each day. Transactions in foreign exchange markets determine the rates at which currencies are exchanged, which in turn determine the costs of purchasing foreign goods and financial assets. Therefore, understanding the evolution of exchange rates will be very important for many outstanding issues in international economics and finance, such as international trade and capital flows, international portfolio management, currency options pricing, and foreign exchange risk management (Hong *et al.*, 2007).

Applying quantitative methods for forecasting in foreign exchange markets and assisting investment decision making has become more indispensable in business practices than ever before. Hence, in today's world, exchange rate modelling and forecasting has become a common research stream. For large multinational firms, which conduct substantial currency transfers in

the course of business, being able to accurately forecast movements of currency exchange rates can result in substantial improvement in the overall profitability of the firm. Over this time, the research stream has gained momentum with the advancement of computer technologies, which have made many elaborate computation methods available and practical. However, the literature shows that predicting the exchange rate movements are largely unforecastable due to their high volatility and noise and still are a problematic task for both academic researchers and business practitioners to make short term and long term forecasting efficiently (Trapletti *et al.*, 2002; Kilian and Taylor, 2003). Artificial neural networks (ANNs) are the most important types of nonparametric nonlinear models, which have been proposed and examined for forecasting exchange rates. Basic notions of the concepts and principals of the ANNs systems and their training procedures can be found in Rumelhart and McClelland (1986), Lippman (1987), and Medsker *et al.* (1993). Artificial neural networks have some advantages over other forecasting models, which make it attractive in exchange rates modelling. First, neural networks

*Corresponding Author Email: khashei@in.iut.ac.ir
Tel.: + 98 9131262551

have flexible nonlinear function mapping capability, which can approximate any continuous measurable function with arbitrarily desired accuracy (Cybenko, 1989; Hornik *et al.*, 1989), whereas most of the commonly used nonlinear time series models in foreign exchange markets do not have this property. Second, being nonparametric and data-driven models, neural networks impose few prior assumptions on the underlying process from which data are generated (Zhang *et al.*, 1998). Because of this property, neural networks are less susceptible to model misspecification problem than most parametric nonlinear methods. This is an important advantage since exchange rate does not exhibit a specific nonlinear pattern. Third, neural networks are adaptive in nature. The adaptivity implies that the network's generalization capabilities remain accurate and robust in a nonstationary environment whose characteristics may change over time. Fourth, neural networks use only linearly many parameters, whereas traditional polynomial, spline, and trigonometric expansions use exponentially many parameters to achieve the same approximation rate (Chakradhara and Narasimhan, 2007).

Given the advantages of neural networks, it is not surprising that this methodology has attracted overwhelming attention in financial markets, and especially exchange rate prediction. Many researchers have investigated the artificial neural networks as models for forecasting exchange rates and have shown that neural networks can be one of the very useful tools in foreign exchange markets forecasting (Zhang *et al.*, 1998). Weigend *et al.* (1991) have found that neural networks are better than random walk models in predicting the Deutsche mark against the US dollar exchange rate. Kuan and Liu (1995) use both feed forward and recurrent neural networks to forecast five foreign exchange rates of the British pound, the Canadian dollar, the Deutsche mark, the Japanese yen, and the Swiss franc against the US dollar. They find that neural networks are able to improve the sign predictions and its forecasts are always better than the random walk forecasts. Hann and Steurer (1996) make comparisons between the neural network and the linear model in US dollar against the Deutsche mark forecasting. They report that if weekly data are used, neural network is much better than both the monetary and random walk models.

Hawley *et al.* (1990) provide an overview of the neural networks in the field of finance. Wu (1995) conducts a comparative study between neural network and ARIMA model in forecasting the Taiwan/US dollar exchange rate. His findings

show that neural network produces significantly better results than the best ARIMA model. Gencay (1999) compares the performance of neural networks with those of random walk and GARCH models in forecasting daily spot exchange rates for the British pound, the Deutsche mark, the French franc, the Japanese yen, and the Swiss franc. He finds that forecasts generated by neural networks are superior to those of random walk and GARCH models. Brooks (1996) document some predictability of daily exchange rates using artificial neural networks.

Leung *et al.* (2000) compare the forecasting accuracy of multilayer perceptron with the general regression neural networks (GRNNs). They show that the GRNN possessed a greater forecasting strength relative to MLFN with respect to a variety of currency exchanges. Santos *et al.* (2007) investigate the hypothesis that the nonlinear mathematical models of multilayer perceptron and the radial basis function neural networks are able to provide a more accurate out-of-sample forecast than the traditional linear models. Their results indicate that ANNs perform better than their linear models. Panda and Narasimhan (Chakradhara and Narasimhan, 2007) compare the forecasting accuracy of neural network with those of linear autoregressive and random walk models in prediction of weekly Indian rupee against the US dollar exchange rate. They report that neural network is much better than both the linear autoregressive and random walk models in out-of-sample forecasting. In contrast to the negative findings of Diebold and Nason, these studies suggest that for high frequency data, the global neural networks approaches may be superior to the local nonparametric approaches.

Despite the all advantages cited for them, neural networks have weaknesses, one of the most important of which is their requirement for large amounts of data in order to yield accurate results (Zhang *et al.*, 1998). No definite rule exists for the sample size requirement of a given problem. The amount of data for network training depends on the network structure, the training method, and the complexity of the particular problem or the amount of noise in the data on hand. Nam and Schaefer (1995) tested the effect of different training sample sizes and found that as the training sample size increased, the ANNs forecaster performed better. With a large enough sample, neural networks can model any complex structure in the data. Hence, neural networks can benefit more from large samples than linear statistical models can. However, in real world situations and in financial markets specifically,

the environment is full of uncertainties and changes occur rapidly; thus, future situations must be usually forecasted using the scant data made available over a short span of time. Therefore, forecasting in these situations requires methods that work efficiently with incomplete data.

Fuzzy forecasting models are suitable under incomplete data conditions and require fewer observations than other forecasting models. Fuzzy theory was originally developed to deal with problems involving linguistic terms (Zadeh, 1975; 1976) and have been successfully applied to various financial markets forecasting (Leon *et al.*, 2006; Yu, 2005). Tanaka *et al.* have suggested fuzzy regression to solve the fuzzy environment and to avoid a modelling error (Tanaka, 1987; Tanka *et al.*, 1982). Time-series models had failed to consider the application of fuzzy theory until Song and Chissom (1993;1994) defined fuzzy time-series. They proposed the definitions of fuzzy time series and methods to model fuzzy relationships among observations. Different fuzzy time-series models have been proposed by following Song and Chissom's definition of fuzzy time series. Although fuzzy forecasting methods can be applied to situations with scant available data and have no data limitation, their performance is not always satisfactory.

Using hybrid models or combining several models has become a common practice to improve forecasting accuracy. The literature on this topic has expanded dramatically since the early work of Reid (1968) and Bates and Granger (1969). Clemen provided a comprehensive review and annotated bibliography in this area (Clemen, 1989). The basic idea of the model combination in forecasting is to use each model's unique feature to capture different patterns in the data. Both theoretical and empirical findings suggest that combining different methods can be an effective and efficient way to improve forecasts (Zhang, 2003). Wedding and Cios (1996) constructed a combination model incorporating radial basis function neural networks and the Box-Jenkins model to time series predict; the experiments proved that the hybrid model for time series forecasting is capable of giving significantly better results. Luxhoj *et al.* (1996) presented a hybrid econometric and neural networks approach for sales forecasting and obtained good prediction performance. Zhang (2003) proposed a hybrid methodology that combined both ARIMA and ANN models taking advantage of the unique strengths of these models in linear and nonlinear modeling for time series forecasting. Empirical results with real data sets indicated that the hybrid

model could provide an effective way to improve the forecasting accuracy achieved by either of the models used separately.

Recently, more hybrid models have been also developed that integrate neural networks with other forecasting models and applied to financial markets forecasting with good prediction performance. Pai and Lin (2005) proposed a hybrid methodology to exploit the unique strength of ARIMA models and Support Vector Machines for stock prices forecasting. Shazly and Shazly (1999) designed a hybrid model combining neural networks and genetic training to the 3-month spot rate of exchange for four currencies: the British pound, the German mark, the Japanese yen, and the Swiss franc. Armano *et al.* (2005) presented a new hybrid approach that integrated artificial neural network with genetic algorithms (GAs) to stock market forecast. Tseng *et al.* (2001) proposed a hybrid model called FARIMA and applied it in Taiwan exchange rate market forecasting. Ricardo and Ferreira (2009) proposed the morphological-rank-linear time-lag added evolutionary forecasting (MRLTAEF) method in order to overcome the random walk dilemma for financial time series prediction. Huang *et al.* (2008) presented a hybrid financial analysis model including static and trend analysis models to construct and train a back-propagation neural network (BPN) model. Lin (2009) proposed a new approach by three kinds of two-stage hybrid models of logistic regression-ANN, to explore if the two-stage hybrid model outperformed the traditional ones, and to construct a financial distress warning system for banking industry. Huang and Jane (2009) proposed a hybrid model by combining the moving average autoregressive exogenous (ARX) prediction model with grey systems theory and rough set (RS) theory to create an automatic stock market forecasting and portfolio selection mechanism. Ahn and Kim (2009) proposed a novel approach to enhance the prediction performance of CBR for the prediction of corporate bankruptcies. Yu *et al.* (2005) proposed a novel nonlinear ensemble forecasting model integrating generalized linear auto regression (GLAR) with artificial neural networks in order to obtain accurate prediction in foreign exchange market. Their findings indicated that the nonlinear ensemble model could be used as an alternative forecasting tool for exchange rates to achieve greater forecasting accuracy. Zhang and Berardi (2001) adopted a different approach, instead of using single network architecture for exchange rate forecasting. Their results indicated that the ensemble network could consistently outperform a

single network design. Tsaih *et al.* (1998) presented a hybrid Artificial Intelligence (AI) approach that integrated the rule-based systems technique and neural networks to S&P 500 stock index prediction. Ince and Trafalis (2006) proposed a two-stage forecasting model which incorporates parametric techniques such as auto-regressive integrated moving average (ARIMA), vector autoregressive (VAR) and co-integration techniques, and nonparametric techniques such as support vector regression (SVR) and artificial neural networks for foreign exchange rate forecasting. Ni and Yin (2009) described a hybrid model formed by a mixture of various regressive neural network models, such as temporal self-organizing maps and support vector regressions, for modelling and prediction of foreign exchange rate. He *et al.* (2010) proposed a hybrid slantlet denoising least squares support vector regression model for exchange rate prediction.

In this paper, the fuzzy logic is used to propose a new hybrid model in order to overcome the data limitation of neural networks and yield more accurate results than traditional neural networks to financial time series forecasting under incomplete data conditions. In proposed model, in first stage, a neural network is used to pre-process raw data and provide the necessary background in order to apply a fuzzy regression model. In second stage, the model parameters are considered in fuzzy number form and optimum values of model parameters of proposed model are calculated using the basic concept of fuzzy regression. To show its appropriateness, our proposed model is applied to exchange rate forecasting problems and their performance is compared with those of some fuzzy and non-fuzzy forecasting models. Empirical results of three cases of exchange rate forecasting - the British pound, the United States dollar, and the Euro against the Iran rial - indicate that the proposed model is an effective way in order to improve forecasting accuracy. The rest of the paper is organized as follows. The neural network modelling approach to time series forecasting are reviewed in Section 2. In Section 3, the proposed model is illustrated. In Section 4, the proposed model is applied to exchange rate forecasting and its performance is compared to some other forecasting models, presented for exchange rate forecasting in incomplete data situations. Finally, the conclusions are discussed.

2. The artificial neural networks (ANNs)

Artificial Neural Networks are flexible computing frameworks for modelling a broad

range of nonlinear problems. One significant advantage of the neural network models over other classes of nonlinear models is that artificial neural networks are universal approximators that can approximate a large class of functions with a high degree of accuracy (Cybenko, 1989; Hornik *et al.*, 1989). Their power comes from the parallel processing of the information from the data. No prior assumption of the model form is required in the model building process. Instead, the network model is largely determined by the characteristics of the data. Single hidden layer feed forward network is the most widely used model form for time series modelling and forecasting. The model is characterized by a network of three layers of simple processing units connected by acyclic links (Fig. 1). The relationship between the output (y_t) and the inputs (y_{t-1}, \dots, y_{t-p}) has the following mathematical representation:

$$y_t = w_0 + \sum_{j=1}^q w_j \cdot g(w_{0,j} + \sum_{i=1}^p w_{i,j} \cdot y_{t-i}) + \varepsilon_t, \quad (1)$$

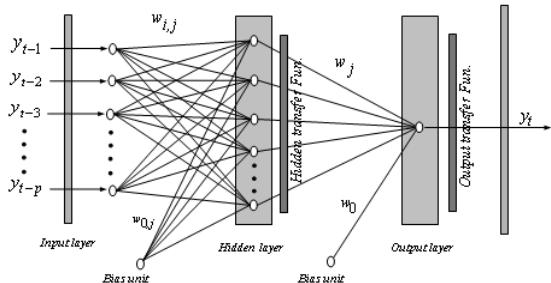
where,

$$w_{i,j} \quad (i = 0, 1, 2, \dots, p, j = 1, 2, \dots, q)$$

and w_j ($j = 0, 1, 2, \dots, q$) are model parameters often called connection weights; p is the number of input nodes; and q is the number of hidden nodes. Hence, the ANN model of (1), in fact, performs a nonlinear functional mapping from past observations to the future value y_t , i.e.,

$$y_t = f(y_{t-1}, \dots, y_{t-p}, w) + \varepsilon_t, \quad (2)$$

where, w is a vector of all parameters and $f(\cdot)$ is a function determined by the network structure and connection weights. Thus, the neural network is equivalent to a nonlinear autoregressive model. Note that expression (1) implies one output node in the output layer, which is typically used for one-step-ahead forecasting. The simple network given by (1) is surprisingly powerful in that it is able to approximate the arbitrary function as the number of hidden nodes when q is sufficiently large (Zhang *et al.*, 1998). In practice, simple network structure that has a small number of hidden nodes often works well in out-of-sample forecasting. This may be due to the over fitting effect typically found in the neural network modelling process. It occurs when the network has too many free parameters, which allow the network to fit the training data well, but typically

Figure 1: Neural network structure ($N^{(p-q-1)}$).

lead to poor generalization. In addition, it has been experimentally shown that the generalization ability begins to deteriorate when the network has been trained more than necessary, that is when it begins to fit the noise of the training data (Morgan and Bourlard, 1990).

Although there exists many different approaches for finding the optimal architecture of a neural network, these methods are usually quite complex in nature and are difficult to implement (Zhang *et al.*, 1998). Furthermore, none of these methods can guarantee the optimal solution for all real forecasting problems. To date, there is no simple clear-cut method for determination of these parameters and the usual procedure is to test numerous networks with varying numbers of input and hidden units (p, q), estimate generalization error for each and select the network with the lowest generalization error (Hosseini *et al.*, 2006). Once a network structure (p, q) is specified, the network is ready for training a process of parameter estimation. The parameters are estimated such that an overall accuracy criterion such as the mean squared error is minimized. This is done with some efficient nonlinear optimization algorithms other than the basic back propagation training algorithm (Rumelhart and McClelland, 1986). The estimated model is usually evaluated using a separate hold-out sample that is not exposed to the training process.

3. Formulation the hybrid model

The neural network model is a precise forecasting model for a broad range of nonlinear problems, but it requires a large amount of historical data to produce accurate results. However, in our society today, due to factors of uncertainty from the integral environment and rapid development of new technologies, we usually have to forecast future situations using little data over a short span of time. Therefore, forecasting models are needed that are efficient under incomplete data conditions. The

fuzzy regression model is the suitable interval-forecasting model for cases where inadequate historical data are available, but its performance is not always satisfactory (Khashei *et al.*, 2008; Khashei *et al.*, 2009).

Many researches in time series forecasting have been argued that using hybrid models or combining several models can be an effective way in order to overcome the limitations of the single models and improve forecasting accuracy. The motivation for combining models comes from the assumption that either one cannot identify the true data generating process or that a single model may not be sufficient to identify all the characteristics of the time series (Hibon and Evgeniou, 2005). Our purpose in this paper is to use the advantages of the fuzzy regression model to eliminate the limitation of a large amount of historical data of neural networks and yield more accurate hybrid model on formulating the proposed model.

The parameters of neural network models (weights and biases) are crisp ($w_{i,j}$ ($i = 0, 1, 2, \dots, p$ $j = 1, 2, \dots, q$), w_j ($j = 0, 1, 2, \dots, q$)).

Instead of using crisp, fuzzy parameters in the form of triangular fuzzy numbers are used for related parameters of layers ($\tilde{w}_{i,j}$ ($i = 0, 1, 2, \dots, p$ $j = 1, 2, \dots, q$), \tilde{w}_j ($j = 0, 1, 2, \dots, q$)). In addition, this study adapts the methodology formulated by Ishibuchi and Tanaka (1988) for the condition which includes a wide spread of the forecasted interval. A proposed model is described using a fuzzy function with a fuzzy parameter:

$$\tilde{y}_t = f(\tilde{w}_0 + \sum_{j=1}^q \tilde{w}_j \cdot g(\tilde{w}_{0,j} + \sum_{i=1}^p \tilde{w}_{i,j} \cdot y_{t-i})) \quad (3)$$

where, y_t are observations, $\tilde{w}_{i,j}$ ($i = 0, 1, 2, \dots, p$, $j = 1, 2, \dots, q$), \tilde{w}_j ($j = 0, 1, 2, \dots, q$) are fuzzy numbers. Equation (3) is modified as follows:

$$\tilde{y}_t = f(\tilde{w}_0 + \sum_{j=1}^q \tilde{w}_j \cdot \tilde{X}_{t,j}) = f(\sum_{j=0}^q \tilde{w}_j \cdot \tilde{X}_{t,j}), \quad (4)$$

where,

$$\tilde{X}_{t,j} = g(\tilde{w}_{0,j} + \sum_{i=1}^p \tilde{w}_{i,j} \cdot y_{t-i}).$$

Fuzzy parameters in the form of triangular fuzzy numbers $\tilde{w}_{i,j} = (a_{i,j}, b_{i,j}, c_{i,j})$ are used:

$$\mu_{\tilde{w}_{i,j}}(w_{i,j}) = \begin{cases} \frac{1}{b_{i,j} - a_{i,j}}(w_{i,j} - a_{i,j}) & \text{if } a_{i,j} \leq w_{i,j} \leq b_{i,j} \\ \frac{1}{b_{i,j} - c_{i,j}}(w_{i,j} - c_{i,j}) & \text{if } b_{i,j} \leq w_{i,j} \leq c_{i,j}, \\ 0 & \text{Otherwise} \end{cases} \quad (5)$$

where, $\mu_{\tilde{w}}(w_{i,j})$ is the membership function of the fuzzy set that represents parameter $w_{i,j}$. Applying the extension principle, it becomes clear that the membership of

$$\tilde{X}_{t,j} = g(\tilde{w}_{0,j} + \sum_{i=1}^p \tilde{w}_{i,j} \cdot y_{t-i})$$

in Equation (4) is given as:

$$\mu_{\tilde{X}_{t,j}}(x_{t,j}) = \begin{cases} \frac{(X_{t,j} - g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i}))}{g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i}) - g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i})} & \text{if I} \\ \frac{(X_{t,j} - g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i}))}{g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i}) - g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i})} & \text{if II} \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

$$I : g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i}) \leq X_{t,j} \leq g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i})$$

$$II : g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i}) \leq X_{t,j} \leq g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i}),$$

where,

$$y_{t,i} = 1(t=1,2,\dots,k \ i=0),$$

$$y_{t,i} = y_{t-i}(t=1,2,\dots,k \ i=1,2,\dots,p)$$

The proof for this equation is given in appendix A.

Considering triangular fuzzy numbers, $\tilde{X}_{t,j}$ with membership function Equation (6) and triangular fuzzy parameters \tilde{w}_j will be as follows:

$$\mu_{\tilde{w}_j}(w_j) = \begin{cases} \frac{1}{e_j - d_j}(w_j - d_j) & \text{if } d_j \leq w_j \leq e_j, \\ \frac{1}{e_j - f_j}(w_j - f_j) & \text{if } e_j \leq w_j \leq f_j, \\ 0 & \text{otherwise,} \end{cases} \quad (7)$$

The membership function of

$$\tilde{y}_t = f(\tilde{w}_0 + \sum_{j=1}^q \tilde{w}_j \cdot \tilde{X}_{t,j}) = f(\sum_{j=0}^q \tilde{w}_j \cdot \tilde{X}_{t,j})$$

is given as:

$$\mu_{\tilde{Y}}(y_t) \equiv$$

$$\begin{cases} \frac{-B_1}{2A_1} + \left[\frac{(B_1)}{2A_1}^2 - \frac{C_1 - f^{-1}(y_t)}{A_1} \right]^{\frac{1}{2}} & \text{if I} \\ \frac{B_2}{2A_2} + \left[\frac{(B_2)}{2A_2}^2 - \frac{C_2 - f^{-1}(y_t)}{A_2} \right]^{\frac{1}{2}} & \text{if II} \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

$$I : C_1 \leq f^{-1}(y_t) \leq C_3 \quad II : C_3 \leq f^{-1}(y_t) \leq C_2$$

where,

$$A_1 = \sum_{j=0}^q (e_j - d_j) \cdot (g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i}) - g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i}))$$

$$B_1 = \sum_{j=0}^q (d_j(g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i}) - g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i})) + (g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i}) \cdot (e_j - d_j)),$$

$$A_2 = \sum_{j=0}^q (f_j - e_j) \cdot (g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i}) - g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i})),$$

$$B_2 = \sum_{j=0}^q (f_j \cdot (g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i}) - g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i})) + g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i}) \cdot (f_j - e_j)),$$

$$C_1 = \sum_{j=0}^q (d_j \cdot g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i}))$$

$$C_2 = \sum_{j=0}^q (f_j \cdot g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i})),$$

$$C_3 = \sum_{j=0}^q (e_j \cdot g(\sum_{i=0}^p b_{i,j} \cdot y_{t,i})),$$

Now considering a threshold level h for all membership function values of observations, the nonlinear programming is given as follows (Khashei and Bijari, 2010):

Min

$$\sum_{t=1}^k \sum_{j=0}^q (f_j \cdot g(\sum_{i=0}^p c_{i,j} \cdot y_{t,i})) - (d_j \cdot g(\sum_{i=0}^p a_{i,j} \cdot y_{t,i}))$$

$$\left\{ \begin{array}{l} \frac{-B_1}{2A_1} + \left[\left(\frac{B_1}{2A_1} \right)^2 - \frac{C_1 - f^{-1}(y_t)}{A_1} \right]^{\frac{1}{2}} \leq h \quad \text{if } I \\ \frac{B_2}{2A_2} + \left[\left(\frac{B_2}{2A_2} \right)^2 - \frac{C_2 - f^{-1}(y_t)}{A_2} \right]^{\frac{1}{2}} \leq h \quad \text{if } II \end{array} \right. \quad (9)$$

$$I: C_1 \leq f^{-1}(y_t) \leq C_3, \quad \text{for } t=1,2,\dots,k$$

$$II: C_3 \leq f^{-1}(y_t) \leq C_2, \quad \text{for } t=1,2,\dots,k$$

As a special case and to present the simplicity and efficiency of the model in forecasting, the triangular fuzzy numbers are considered symmetric, output neuron transfer function is considered to be linear, and connected weights between input and hidden layer are considered to be of a crisp form. The membership function of y_t in the special case mentioned is transformed as follows:

$$\mu_{\bar{y}}(y_t) = \begin{cases} 1 - \frac{|y_t - \sum_{j=0}^q \alpha_j \cdot X_{t,j}|}{\sum_{j=0}^q |X_{t,j}|} & \text{for } X_{t,j} \neq 0, \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

Simultaneously, y_t represents the t th observation and h -level is the threshold value representing the degree to which the model should be satisfied by all the data points y_1, y_2, \dots, y_k . A choice of the h value influences the widths of the fuzzy parameters:

$$\mu_{\bar{y}}(y_t) \geq h \quad \text{for } t=1,2,\dots,k, \quad (11)$$

The index t refers to the number of non-fuzzy data used for constructing the model. On the other hand, the fuzziness S included in the model is defined by:

$$S = \sum_{j=0}^q \sum_{t=1}^k c_j |w_j| |X_{t,j}|, \quad (12)$$

where, w_j is the connection weight between output neuron and j th neuron of the hidden layer; $x_{t,j}$ is the output value of j th neuron of the hidden layer in time t . Next, the problem of finding the parameters in the proposed method is formulated as a linear programming problem:

Minimize

$$S = \sum_{j=0}^q \sum_{t=1}^k c_j |w_j| |X_{t,j}|$$

subject to

$$\left\{ \begin{array}{l} \sum_{j=0}^q \alpha_j X_{t,j} + (1-h)(\sum_{j=0}^q c_j |X_{t,j}|) \geq y_t \quad t=1,\dots,k \\ \sum_{j=0}^q \alpha_j X_{t,j} - (1-h)(\sum_{j=0}^q c_j |X_{t,j}|) \leq y_t \quad t=1,\dots,k \\ c_j \geq 0 \quad \text{for } j=0,1,\dots,q \end{array} \right. \quad (13)$$

The procedure of the proposed model works as follows:

Phase I: Training a network using the available information from observations, i.e., input data is considered to be non-fuzzy. The results from phase I, the optimum solution of the parameter

$$w^* = (w_j^* \ (j=0,1,\dots,q)),$$

$$w_{i,j}^* \ (i=0,1,\dots,p, \ j=0,1,\dots,q)$$

and the output values of the hidden neurons, are used as one of the input data sets in phase II.

Phase II: Determining the minimal fuzziness using the same criterion as in Eq. (13) and

$$w^* = (w_j^* \ (j=0,1,\dots,q),$$

$$w_{i,j}^* \ (i=0,1,\dots,p, \ j=0,1,\dots,q))$$

The numbers of constraint functions are the same as the number of observations.

Phase III: The data around the model's upper bound and lower bound when the proposed model has outliers with a wide spread are deleted in accordance with Ishibuchi's recommendations. In order to make the model to include all possible conditions, C_j has a wide spread when the data set includes a significant difference or outlying case. Ishibuchi and Tanaka (1988) suggest that the data around the model's upper and lower boundaries be

deleted so that the fuzzy regression model can be reformulated.

4. Application the hybrid model to exchange rate forecasting

In this section, three daily data sets of exchange rates—the British pound, the United States dollar, and the Euro against the Iran rial—are used in order to demonstrate the appropriateness and effectiveness of the proposed model. These data sets are collected from central bank of Islamic Republic of Iran (CBI). Only the one-step-ahead forecasting is considered. Different performance indicators such as MAE (Mean Absolute Error), MSE (Mean Squared Error), SSE (Sum Squared Error), RMSE (Root Mean Squared Error), MAPE (Mean Absolute Percentage Error), and ME (Mean Error) are employed in order to measure forecasting performance of proposed model.

4.1. The exchange rate (*British pound / Iran rial*) forecasts

The information used in this investigation consists of 42 daily observations of the exchange rate of the British pound against the Iran rial from *5 November to 16 December, 2005* that are shown in Figure 2. Applying the hybrid method, 35

observations are first used to formulate the model and the last seven observations are used to evaluate the performance of the proposed model.

Phase I: Training neural network model: In order to obtain the optimum network architecture, based on the concepts of artificial neural networks design and using constructive algorithm (Khashei, 2005) in *MATLAB 7* package software, different network architectures are evaluated in order to compare the neural network performance.

The best fitted network which is selected, and therefore, the architecture which presented the best forecasting accuracy with the test data, is composed of two inputs, three hidden and one output neurons (in abbreviated form, $N^{(2-3-1)}$) in which sigmoid and linear transfer functions are used in the hidden and output layers, respectively. Actual and neural network estimated values are shown in Figure 3. The performance measures of the mentioned network are given in Table 1.

Phase II: Determining the minimal fuzziness: Setting

$$(w_0, w_1, w_2, w_3) = (-11.7115, -5.2798, 0.50964, 17.275)'$$

the fuzzy parameters are obtained from Equation (13) (with $h=0$). These results are plotted in Figure 4.

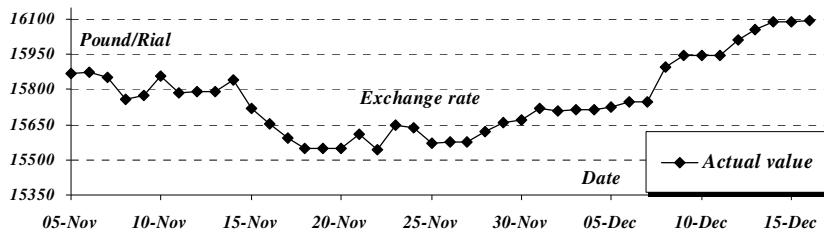


Figure 2: Exchange rate (British pound / Iran rial) from 5 Nov. to 16 Dec., 2005.

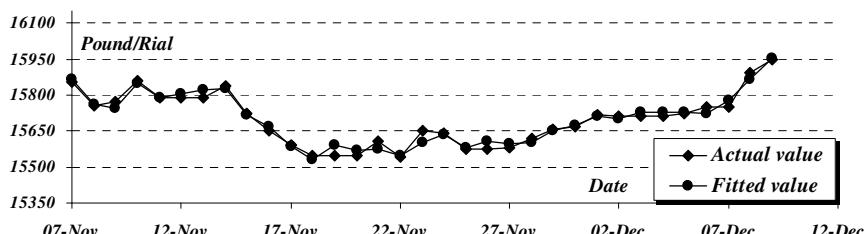


Figure 3: Actual and neural network fitted values.

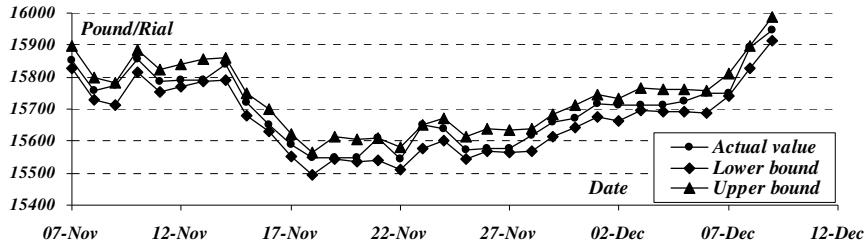


Figure 4: Results obtained from the proposed model.

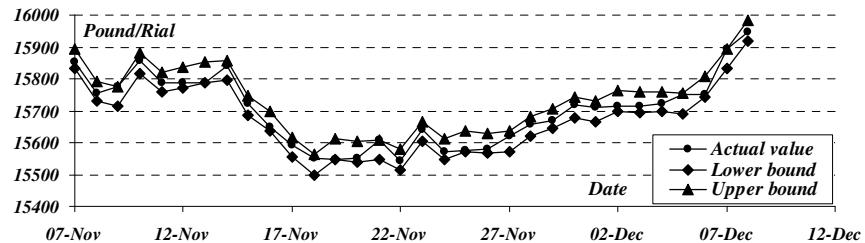


Figure 5: Results of the proposed model after deleting the 8 Dec. upper bound.

Table 1: Performance measures of the designed network, $N^{(2-3-1)}$.

Train				Test			
MSE	MAE	MSE	SSE	RMSE	ME	MAPE	MAE
389.7	15.5	415.7	2909	20.389	1.8	0.1129	18.1

Phase III: It is known from the above results that the observation of *8 December* is located at the upper bound, so the LP constrained equation that is produced by this observation is deleted to renew phase II. The results are shown in Fig. 5. Using the revised proposed model, the future values of the exchange rate for the next seven transaction days are forecasted; the results are shown in Table 2. The results of the forecast are satisfactory, and the fuzzy intervals are narrower than the results before the model was revised.

4.2. The exchange rate (United States dollar / Iran rial) forecasts

The information used in this investigation consists of 42 daily observations of the exchange rate of United States dollar against Iran rial from *5 November to 16 December, 2005* that are shown in Fig. 6. As in the previous section, by applying the hybrid method, 35 observations are first used to formulate the model and the last seven observations are used to evaluate the performance of the proposed model.

Phase I: Training neural network model: Similar to the previous example in order to obtain the optimum network architecture, the best-fitted network is composed of three inputs, three hidden and one output neurons ($N^{(3-3-1)}$) in which sigmoid

and linear transfer functions are used in the hidden layer and the output layer, respectively. Actual and neural network fitted values are shown in Figure 7. The performance measures of the mentioned network are given in Table 3.

Phase II: Determining the minimal fuzziness: Setting

$$(w_0, w_1, w_2, w_3) = (-985.2, 0.5847, 985.3, 0.2781)$$

the fuzzy parameters are obtained from Equation (13) (with $h=0$). These results are plotted in Figure 8.

Phase III: As in the previous example, it is known from the above results that the observation of *9 November* is located at the lower bound, so the LP constrained equation that is produced by this observation is deleted to renew phase II. The results are shown in Figure 9. Using the revised proposed model, the future values of the exchange rate for the next seven transaction days are forecasted; the results are shown in Table 4. The results of the forecast are satisfactory, and the fuzzy intervals are narrower than the results before the model was revised.

Table 2: Results of the proposed model for the test data.

Date	Actual	Lower bound	Upper bound
10 December	15946	15950	16013
11 December	15948	15940	16003
12 December	16013	15981	16045
13 December	16055	16033	16096
14 December	16087	16042	16105
15 December	16092	16043	16106
16 December	16096	16042	16105

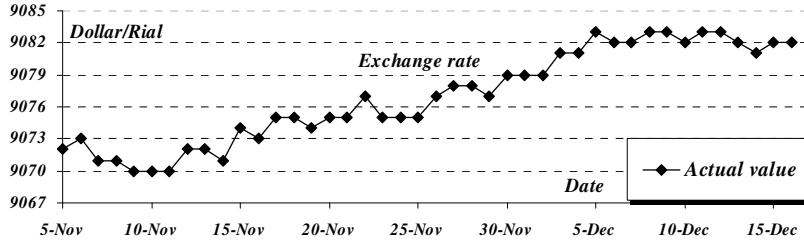


Figure 6: Exchange Rate (US dollar / Iran rial) from 5 Nov. to 16 Dec., 2005.

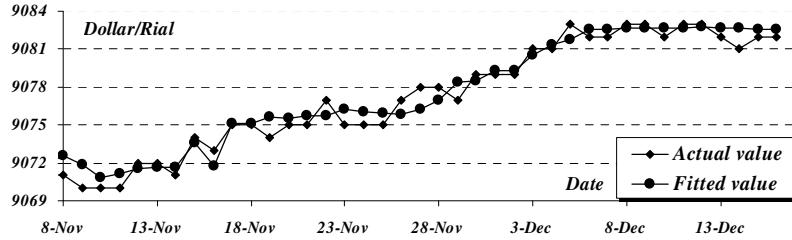


Figure 7: Actual and neural network fitted values.

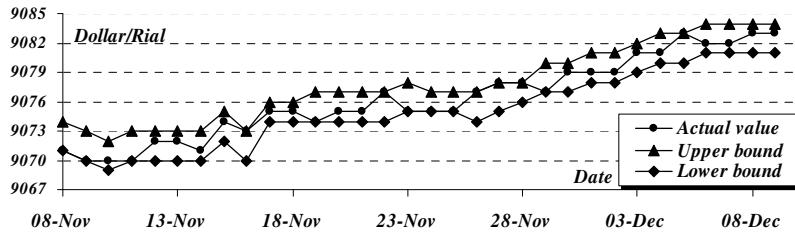


Figure 8: Results obtained from the proposed model.

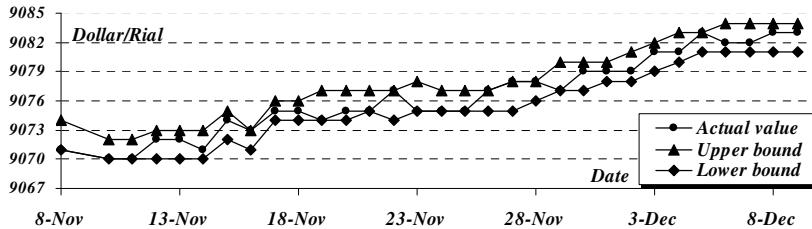


Figure 9: Results of the proposed model after deleting the 9 Nov. lower bound.

Table 3: Performance measures of the designed network, $N^{(3-3-1)}$.

Train			Test				
MSE	MAE	MSE	SSE	RMSE	ME	MAPE	MAE
0.8252	0.9322	0.6859	4.8016	0.8282	-0.5387	0.0076	0.6923

Table 4: Results of the proposed model for the test data.

Date	Actual	Lower bound	Upper bound
10 Dec	9082	9081	9084
11 Dec	9083	9082	9084
12 Dec	9083	9082	9084
13 Dec	9082	9081	9084
14 Dec	9081	9081	9084
15 Dec	9082	9081	9084
16 Dec	9082	9081	9084

Although the proposed model is specifically proposed for forecasting situations with scant historical data available, the performance of the

proposed model can be improved with larger data sets, as is the case with other quantitative forecasting models (Khashei and Bijari, 2010).

4.3. The comparison with other models

In this section, the predictive capabilities of the proposed model are compared with neural network using three example cases studied of exchange rate forecasting (the British pound, the US dollar, and the Euro against the Iran rial). Some other fuzzy and non-fuzzy forecasting models such as Chen's fuzzy time series (first-order) (Chen, 1996), Chen's fuzzy time series (high-order) (Chen and Chung, 2006), Yu's fuzzy

time series (Yu, 2004), Fuzzy Auto Regressive Integrated Moving Average (FARIMA) (Tseng *et al.*, 2001), and ARIMA models have respectively been considered for comparison with the forecasting power of the proposed model in interval and point estimation cases. To measure forecasting performance in the point estimation case, MAE and MSE are employed as performance indicators, which are computed from the following equations, respectively:

$$MAE = \frac{1}{N} \sum_{i=1}^N |e_i| \quad (14)$$

$$MSE = \frac{1}{N} \sum_{i=1}^N (e_i)^2 \quad (15)$$

where e_i is the individual forecast error and N is the number of error terms.

Based on the results obtained from these cases studied (Table 5), the predictive capabilities of the proposed model are rather encouraging and the possible interval by the proposed model is narrower than 95% of the confidence interval of neural network. The width of the forecasted interval in the proposed model is 63.2, 2.5, and 13.5 rials in the British pound, the United States dollar, and the Euro against the Iran rial exchange rate forecasting cases; respectively, which indicate an 88.9%, 84.6%, and 79.8% improvement upon the 95% of the confidence interval of neural network. However, the proposed model requires fewer observations than neural network require and is an interval forecaster that yields more

information. Although the proposed mode is basically designed for interval forecasting, its performance in point estimation is also more satisfactory than that of neural network. For example in terms of MAE, the percentage improvements of the proposed model over the neural network is 3.8%, 15.8%, and 8.8% in the British pound, the United States dollar, and the Euro against the Iran rial exchange rate forecasting cases; respectively. These evidences show that the performance of the proposed model is better than neural network for situations of incomplete data.

Similarity, the possible interval given by the proposed model is also narrower than the interval obtained by FARIMA model. It can be seen that the performance of the proposed model is superior to FARIMA model (45.7%, 40.5%, and 43.0% improvement compared to FARIMA model in the British pound, the United States dollar, and the Euro against the Iran rial exchange rate forecasting cases; respectively. In addition, the performance of the proposed model is better than the Chen's fuzzy time-series (first- and second- order), Yu's fuzzy time-series, and ARIMA models in point estimation applications. According to the numerical results (Table 7), the MAE and MSE of the proposed model are lower than mentioned models in all exchange rate forecasting cases, except of the MSE of the Yu's model in exchange rate (the Euro against the Iran rial) forecasting case.

In addition, fuzzy time series models are methods of point estimation, and it is expected that fuzzified historical data must lose some information in such cases.

Table 5: Comparison of forecasted interval widths by the proposed model compared to other models (interval estimation).

Model	Cases	Forecasted interval width	Related Performance	
			Neural network (95% Confidence)	Fuzzy ARIMA
Neural network (95% Confidence)	British pound	567.9	0	—
		116.4	79.5%	0
		63.2	88.9%	45.7%
Neural network (95% Confidence)	US dollar	16.2	0	—
		4.2	74.1%	0
		2.5	84.6%	40.5%
Neural network (95% Confidence)	Euro	66.9	0	—
		23.7	64.6%	0
		13.5	79.8%	43.0%

Table 6: Comparison of the performance of the proposed model with those of other forecasting models (point estimation).

Model	British Pound		US Dollar		Euro	
	MAE	MSE	MAE	MSE	MAE	MSE
Auto-Regressive Integrated Moving Average (ARIMA)	19.772	849.5	0.924	1.24	56.44	3443
Chen's fuzzy time series (first-order)	29.047	1232.7	0.750	0.777	48.60	3324
Chen's fuzzy time series (second-order)	28.011	1110.3	0.750	0.777	48.60	3324
Yu's fuzzy time series	25.964	917.3	0.750	0.777	40.27	1783
Artificial Neural Network (ANN)	18.112	415.7	0.692	0.686	43.1	2173
Proposed Model	17.429	413.9	0.583	0.524	39.3	2055

Table 7: Improvement percentage of the proposed model in comparison with those of other forecasting models.

Model	British Pound (%)		US Dollar (%)		Euro (%)	
	MAE	MSE	MAE	MSE	MAE	MSE
Auto-Regressive Integrated Moving Average (ARIMA)	11.9	51.3	36.9	57.7	30.4	40.3
Chen's fuzzy time series (first-order)	40.0	66.4	22.3	32.6	19.1	38.2
Chen's fuzzy time series (second-order)	37.8	62.7	22.3	32.6	19.1	38.2
Yu's fuzzy time series	32.9	54.9	22.3	32.6	2.4	-15.3
Artificial Neural Networks (ANNs)	3.8	0.4	15.8	23.6	8.8	5.4

5. Conclusion

The foreign exchange markets are among the most important and the largest financial markets in the world with trading taking place twenty-four hours a day around the globe and trillions of dollars of different currencies transacted each day. Being able to accurately forecast the movements of exchange rates can result in considerable improvement in the overall profitability of the multinational financial firm, especially for firms, conducting substantial currency transfers in the course of business. However, predicting currency movements has always been a problematic task for academic researchers and despite the paramount modelling effort registered in the last three decades, it is widely recognized that exchange rates are extremely difficult to forecast. That is the reason why research on improving the effectiveness of time series models has been never witnessed a halt. Many empirical studies including several large-scale forecasting competitions with a large number of commonly used time series forecasting models conclude that combining forecasts obtained from more than one model often leads to improved performance.

In this paper, a new hybrid model is proposed drawing upon the basic concepts of the artificial neural network and fuzzy regression models. In proposed model, the fuzzy numbers and fuzzy logic are applied in order to overcome the data limitation of the neural network and provides a more flexible and more accurate hybrid model for financial time series forecasting.

Experimental results of exchange rates forecasting indicate that the proposed model requires fewer observations than does neural network to obtain accurate results. The proposed model also obtains narrower possible intervals than other interval-forecasting models do under incomplete data conditions. From the empirical results, it can be seen that the proposed model cannot only make good forecasts in both point and interval estimation, but also provides the decision makers with the best and worst possible situations. So, the proposed model can be used as an alternative forecasting tool for financial markets forecasting, especially in cases where inadequate

historical data are available. In general, the proposed model can be a more appropriate tool:

- (i) when is needed to obtain more accurate results under incomplete data situations .
- (ii) when is needed to collect more historical data in order to yield desired accuracy.
- (iii) when is needed to provide the best and the worst possible situations for decision-making.

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