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Technical Note:

A mathematical model for optimization of strength of concrete: A case study for shear modulus of Rice Husk Ash Concrete

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Abstract

Rice Husk Ash (RHA) is natural Pozzolan containing reactive silica and/or aluminum. When the material is mixed with lime in powdered form and in the presence of water, it will set and harden like cement. This work uses Osadebe's optimization model to optimize the shear modulus of concrete made from RHA. The strengths predicted by the model are in good agreement with their corresponding experimentally obtained values. With the model, any desired strength of hardened concrete, given any mix proportions, is easily evaluated. The average Poisson ratio and mean shear strength for the concrete are found to be 0.26 and 5.5 N/mm^2 respectively.

Keywords: Shear modulus; Poisson ratio; Optimization; Rice-Husk-Ash; Pozzolan; Regression equation

1. Introduction

Concrete, a product of water, cement and aggregate, when sufficiently hardened, is used in various forms to resist load. The cost of one of its ingredients, Ordinary Portland Cement (OPC) is rising rapidly. Cheap and replaceable or complimentary substitutes are being developed [20].

Rice Husk Ash (RHA) is an agro-waste material, found in abundance in Nigeria and in many other parts of the world. RHA is one of the natural Pozzolanas. Pozzolanas are materials containing reactive silica and/or aluminum. When the material is mixed with lime in powdered form and in the presence of water, it will set and harden like cement [4]. The Indian ITDG [16], has it that Greeks and the Romans were the first civilization known to have used Pozzolanas in lime mortars. Udeala [23], produced RHA with 45 percent slaked lime mix. The IS 4098 [15], stipulates specific characteristics of different grades of lime-pozzolan mixture. The strength and other properties are affected by lime-pozzolan ratio. The shear modulus of concrete (G_c) is given by:

$$G_c = \frac{E_c}{2(\mu + 1)} \tag{1}$$

But

$$\mu = \frac{\sigma_T}{\sigma_c} \tag{2}$$

where E_c is the modulus of elasticity of concrete over the linear range of deformation, σ_T is tensile stress at first cracking in flexure and σ_c is compressive stress at first cracking in compression specimen [20]. For normal aggregate concrete with OPC, Poisson ratio varies from 0.3 for low strength concrete to 0.15 for high strength concrete [20].

2. Osadebe's concrete optimization model

According to Osadebe's model [21], concrete is a four-component material, manufactured by mixing water, cement, sand and coarse aggregate. These ingredients are mixed in rational proportions to achieve desired strength of the hardened concrete.

Let us consider an arbitrary amount 'S' of a given concrete mixture. Let the portion of the ith component of the four constituent materials of the concrete be S_i, i = 1, 2, 3, 4. Then, in keeping with the principle of absolute volume (Mass):

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$$S_1 + S_2 + S_3 + S_4 = S \quad (3a)$$

or

$$S_1 / S + S_2 / S + S_3 / S + S_4 / S = 1 \quad (3b)$$

where S_i / S is the proportion of the ith constituent component of the considered concrete mixture. Let:

$$S_i / S = Z_i \quad i = 1, 2, 3, 4 \quad (4)$$

Substituting Equation (4) into Equation (3b), we have:

$$Z_1 + Z_2 + Z_3 + Z_4 = 1 \quad (5)$$

where Z_1, Z_2, Z_3, and Z_4 are proportions of water, cement, sand and coarse aggregate respectively.

In general, for any given concrete mixture, exists a vector Z (Z_1, Z_2, Z_3, Z_4) whose elements satisfies Equation 5. In addition, for each Z_i, the following inequality holds:

$$Z_i > 0 \quad (6)$$

It is widely known that the proportions of its relative constituent ingredients govern the strength of hardened concrete. In other words, there exist a wide variety of strengths of hardened concrete in relation to the mix proportions of its constituent components: water, cement, sand and coarse aggregate. Consequently the compressive strength, y, of concrete can be expressed in a mathematical term as:

$$Y = f\,(Z_1, Z_2, Z_3, Z_4)$$

where $f\,(Z_1, Z_2, Z_3, Z_4)$ is a multi-variate response function whose variables Z_i are subject to the constraints as defined in Equations (5) and (6).

2.1. The regression equation

On the assumption that the response function is continuous and differentiable with respect to its variables, Z_i, it can be expanded in Taylor's series in the neighborhood of a chosen point $Z(0) = (Z^{(0)}_1, Z^{(0)}_2, Z^{(0)}_3, Z^{(0)}_4)^T$ as follows:

$$f\,(Z) = f\,(Z^{(0)} + \sum_{i=1}^{4} \frac{\partial f\,(Z^{(0)})}{\partial Z_i}(Z_i - Z_i^{(0)})$$

$$+ \tfrac{1}{2}! \sum_{i=1}^{3} \sum_{j=1}^{4} \frac{\partial^2 f\,(Z^{(0)})}{\partial Z_i Z_j}(Z_i - Z_i^{(0)})(Z_j - Z_j^{(0)})$$

$$+ \tfrac{1}{2}! \sum \frac{\partial^2 f\,(Z^{(0)})}{\partial Z_i^2}(Z_i - Z_i^{(0)})^2 + ... \quad (7)$$

For convenience, the point $Z^{(0)}$ can be chosen to be the origin without loss of generality of the formulation. Consequently, $Z^{(0)} = 0$, implies that $Z_1^{(0)} = 0$, $Z_2^{(0)} = 0$, $Z_3^{(0)}$ and $Z_4^{(0)} = 0$. Let:

$$b_0 = f\,(0), \qquad b_i = \frac{\partial f\,(0)}{\partial z_i}, \qquad b_{ij} = \frac{\partial^2 f\,(0)}{\partial z_i \partial z_j},$$

$$b_{ii} = \frac{\partial^2 f\,(0)}{\partial z_i^2}$$

Equation (7) can then be written as follows:

$$f(z) = b_o + \sum_{i=1}^{4} b_i z_i + \sum_{i=1}^{3} \sum_{i=1}^{4} b_{ij} z_i z_j + \sum_{i=1}^{4} b_{ii} z_i^2 + ... \quad (8)$$

The number of constant coefficients N of the above polynomial (Equation 8) is given by:

$$N = C_m^{m+q} \quad (9)$$

where m is the degree of the polynomial of the1 response function and q is the number of variables, here $Z=4$. However, taken advantage of Equation (5), the number of coefficients can be reduced to:

$$N = C_m^{m+q-1} \text{ or } C_m^{m+3} \quad (10a)$$

But

$$C_m^{m+q-1} = \frac{q(q+1)(q+2)...(q+m+1)}{m!} \quad (10b)$$

Multiplying Equation (5) by b_0 can do reduction of Equation 8 to equivalent polynomial with less number of coefficients. Thus we have:

$$Z_1 + b_0 Z_2 + b_0 Z_3 + b_0 Z_4 = b_0 \quad (11)$$

Again multiplying Equation 5 by Z_1, Z_2, Z_3, and Z_4 in succession and making Z_1^2, Z_2^2, Z_3^2 and Z^2_4 subject of formula and rearranging, we have:

$$Z_1^{\,2} = Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4 \quad (12a)$$

$$Z_2^{\,2} = Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4 \quad (12b)$$

$$Z_3^{\,2} = Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4 \quad (12c)$$

$$Z_4^{\,2} = Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4 \quad (12d)$$

Substitution of Equations (11) and (12a) to 12(d) into Equation (8) gives, in expanded form, the following expression:

$$\begin{aligned} Y =\ & b_0Z_1 + b_0Z_2 + b_0Z_3 + b_0Z_4 + b_1Z_1 + b_2Z_2 \\ & + b_3Z_3 + b_4Z_4 + b_{12}Z_1Z_2 + b_{13}Z_1Z_3 \\ & + b_{14}Z_1Z_4 + b_{23}Z_2Z_3 + b_{24}Z_2Z_4 + b_{34}Z_3Z_4 \\ & + b_{11}(Z_1 - Z_1Z_2 - Z_1Z_3 - Z_1Z_4) \\ & + b_{22}(Z_2 - Z_1Z_2 - Z_2Z_3 - Z_2Z_4) \\ & + b_{33}(Z_3 - Z_1Z_3 - Z_2Z_3 - Z_3Z_4) \\ & + b_{44}(Z_4 - Z_1Z_4 - Z_2Z_4 - Z_3Z_4) \end{aligned} \quad (13a)$$

Factorization of Equation (13a) gives:

$$\begin{aligned} Y =\ & (b_0 + b_1 + b_{11})Z_1 + (b_0 + b_2 + b_{22})Z_2 \\ & + (b_0 + b_3 + b_{33})Z_3 + (b_0 + b_4 + b_{44})Z_4 \\ & + (b_{12} - b_{11} - b_{22})Z_1Z_2 \\ & + (b_{13} - b_{11} - b_{33})Z_1Z_3 \\ & + (b_{14} - b_{11} - b_{44})Z_1Z_4 \\ & + (b_{23} - b_{22} - b_{33})Z_2Z_3 \\ & + (b_{24} - b_{22} - b_{44})Z_2Z_4 \\ & + (b_{34} - b_{33} - b_{44})Z_3Z_4 \end{aligned} \quad (13b)$$

Defining $\beta_i = b_0 + b_i + b_{ii}$ & $\beta_{ij} = b_{ij} + b_{ii} + b_{jj}$, $i, j = 1,2,3,4,$ Equation (13b) becomes:

$$\begin{aligned} Y =\ & \beta_1Z_1 + \beta_2Z_2 + \beta_3Z_3 + \beta_4Z_4 + \beta_{12}Z_1Z_2 \\ & + \beta_{13}Z_1Z_3 + \beta_{14}Z_1Z_4 + \beta_{23}Z_2Z_3 + \beta_{24}Z_2Z_4 \\ & + \beta_{34}Z_3Z_4 \end{aligned} \quad (14a)$$

or

$$Y = \sum_{i=1}^{4} \beta_i Z_i + \sum_{1 \le i \le j \le 4} \beta_{ij} Z_i Z_j \quad (14b)$$

Equations (13) and (14) are equivalent, only that the coefficients of Equation (13) are fifteen in number while those of Equation (14) are ten. Equation (14) is the regression equation.

The response function is defined if the values of the unknown constant coefficients β_i and β_{ij} are uniquely determined.

On the other hand, these coefficients are determined if the values of the response function are known for (N= 10) different points on the response surface through experimental observations (measurements).

2.2. The coefficients of the regression equation

Let the Kth response (compressive strength for the serial number *k*) be $y^{(k)}$ and the vector of the corresponding set of variables be (see Table 1):

$$Z^{(k)} = [Z_1^{(k)}, Z_2^{(k)}, Z_3^{(k)}, Z_4^{(k)}]^T$$

Substitution of the above vector in Equation (14) for $k = 1, 2, \ldots, 10,$ generates the following system of ten linear algebraic equations in the unknown coefficients β_i and β_{ij}.

$$Y^{(k)} = \sum_{i=1}^{4} \beta_i Z_i^{(k)} + \sum_{1 \le i \le j \le 4} \beta_{ij} Z_i^{(k)} Z_j^{(k)}$$

$$k = 1, \ldots, 10 \quad (15)$$

Let:

$$[y^{(k)}] = \begin{bmatrix} y(1) \\ y(2) \\ . \\ . \\ . \\ y(10) \end{bmatrix}$$

$$[Z] = \begin{bmatrix} Z_1^{(1)} & Z_1^{(2)} & \ldots & Z_1^{(10)} \\ Z_2^{(1)} & Z_2^{(2)} & \ldots & Z_2^{(10)} \\ Z_3^{(1)}Z_4^{(1)} & Z_3^{(2)}Z_4^{(2)} & \ldots & Z_3^{(10)}Z_4^{(10)} \end{bmatrix}$$

and

$$[B] = [\beta_1, \beta_1, \ldots, \beta_{34}]$$

The explicit matrix form of Equation (15) can be written as:

$$[y^{(k)}] = [B][Z] \qquad (16a)$$

Since the vector (Z) values are known (easily determined), we can re-arrange (16a) as:

$$[Z]^T [B]^T = [Y^{(k)}] \qquad (16b)$$

Solution of Equation (16b) gives the values of the unknown coefficients of the regression equation. The matrix Z^T based on Table 1 is shown in Table 2

3. Materials and methods

The main material for this research is the Rice Husk Ash (RHA)-slaked lime mix.

The mix ratios used for the simplex design points were as a result of preliminary research findings about the concrete made from the Pozzolan.

3.1. Preparation of samples

a. The RHA was used as supplied,
b. Aggregates,
i. Sand.

The sand was collected from River Benue, Makurdi-Nigeria. It was prepared to

BS 1017: parts 1 and 2 [14] and BS 882: [8]. The grading was carried out to BS 812:103: [6]. The sand belongs to grading zone C [20].

Coarse aggregate (crushed granite).The crushed granite chippings were collected from Kwande, Benue State-Nigeria. The maximum size of aggregate used was 20mm.

3.2. Poisson Ratio and Shear modulus of the concrete

Concrete cylinders of size 150 diameter by 300mm height (length) were cast from pre-determined proportions of water, cement, sand and crushed granite chippings according to BS 1881:part 110: [9]. The cylinders were demoulded after 3 days (72 hours) and immediately transferred to the curing tank at room temperature for 56 days.

The cylinders were then tested for compression and tensile strengths according to BS 1881: part 116 [12] and part 117 [13] respectively. The Poisson ratio is calculated using Equation (2) while the shear strength is calculated using Equation (1).

4 Results and analysis

4.1 Poisson ratio

The results of the Poisson ratio test are shown in Table 3.

4.2 Shear modulus test results, based on Osadebe's second-degree polynomial.

The results of the Shear modulus test results, based on Osadebe's [21] second-degree polynomial are shown in Table 4.

Legend:

$$\breve{y} = \frac{\sum_{i=1}^{m} y_i}{m_i}$$

$$S_i^2 = \frac{1}{m_{i-1}} \left[\sum_{r=1}^{mi} y_r^2 - \frac{1}{m_i} \left(\sum_{r=1}^{m_i} y_r\right)^2 \right]$$

Table 1. Selected mix ratios and component's fraction based on Osadebe's second-degree polynomial.

S/N	Mix Ratios				Component's Fraction			
	S_1	S_2	S_3	S_4	Z_1	Z_2	Z_3	Z4
1	0.88	1	2½	4	0.1050	0.1193	0.2983	0.4773
2	0.86	1	2	4	0.1094	0.1272	0.2545	0.5089
3	0.855	1	2	3½	0.1162	0.1360	0.2719	0.4759
4	0.86	1	2	3	0.1254	0.1458	0.2915	0.4373
5	0.855	1	2½	3½	0.1088	0.1273	0.3183	0.4456
6	0.865	1	3	4	0.0976	0.1128	0.3384	0.4512
7	0.87	1	3	4½	0.0929	0.1067	0.3202	0.4803
8	0.86	1	1½	3	0.1351	0.1572	0.2358	0.4717
9	0.86	1	2¾	3 2/5	0.1074	0.1248	0.3433	0.4245
10	0.865	1	2	4¼	0.1066	0.1232	0.2465	0.5237
Control								
11	0.858	1	2 3/7	4	0.1036	0.1207	0.2931	0.4827
12	0.86	1	1¾	3	0.1301	0.1513	0.2648	0.4539
13	0.855	1	2 2/5	3½	0.1103	0.1289	0.3095	0.4513
14	0.86	1	2	4 1/3	0.1050	0.1221	0.2441	0.5289
15	0.862	1	2¼	3 1/8	0.1191	0.1382	0.3109	0.4318
16	0.858	1	2	2 5/6	0.1282	0.1495	0.2989	0.4234
17	0.858	1	2 2/3	3 2/7	0.1129	0.1314	0.3505	0.4318
18	0.86	1	3	4 1/8	0.0957	0.1113	0.3339	0.4730
19	0.855	1	2	3	0.1247	0.1459	0.2918	0.4376
20	0.8595	1	2 ¾	4	0.0998	0.1162	0.3194	0.4646

Table 2. Z^T matrix, based on Table 1.

Z_1	Z_2	Z_3	Z_4	Z_1Z_2	Z_1Z_3	Z_1Z_4	Z_2Z_3	Z_2Z_4	Z_3Z_4
0.1050	0.1193	0.2983	0.4773	0.01253	0.03132	0,05012	0.03559	0.05694	0.14238
0.1094	0.1272	0.2545	0.5089	0.01139	0.02784	0.05567	0.03237	0.06473	0.12952
0.1162	0.13600.	0.2719	0.4759	0.01580	0.03159	0.05530	0.03698	0.06472	0.12940
0.1254	0.1458	0.2915	0.4373	0.01828	0.03655	0.05484	0.04250	0.06376	0.12747
0.1088	0.1273	0.3183	0.4456	0.01385	0.03463	0.04848	0.04052	0.05672	0.14183
0.0976	0.1128	0.3384	0.4512	0.01101	0.03303	0.04404	0.03817	0.05090	0.15269
0.0929	0.1067	0.3202	0.4803	0.00991	0.02975	0.04462	0.03417	0.05125	0.15379
0.1351	0.1572	0.2358	0.4717	0.02124	0.03186	0.06373	0.03707	0.07415	0.11123
0.1074	0.1248	0.3433	0.4245	0.01340	0.03687	0.04559	0.04284	0.05298	0.14573
0.1066	0.1232	0.2465	0.5237	0.01313	0.02628	0.05583	0.03037	0.06452	0.12909

Table 3. The results of the poisson ratio test.

Expt. No.	Compressive Strength (σ_c) N/mm^2	Tensile Strength (σ_T) N/mm^2	Poisson's Ratio
1	0.57	0.23	0.40
2	0.79	0.23	0.29
3	0.68	0.23	0.34
4	1.02	0.23	0.23
5	1.58	0.36	0.23
6	1.70	0.34	0.20
7	1.36	0.30	0.22
8	2.38	0.35	0.15
9	1.24	0.28	0.23
10	0.79	0.23	0.29
		Σ	**2.58**

Table 4. The results of the Shear modulus test results, based on Osadebe's second-degree polynomial.

Expt. No.	Replicat-ion	Response N/mm^2	Response Symbol	ΣY_i	$\check{y}$	$(\Sigma y_i)^2$	S_i^2
1	1A 1B	3.32 3.34	Y_1	6.66	3.33	44.36	-0.0020
2	2A 2B	4.29 4.20	Y_2	8.49	4.25	72.08	0.0041
3	3A 3B	3.72 3.71	Y_3	7.43	3.72	55.21	-0.0025
4	4A 4B	2.64 2.64	Y_4	5.28	2.64	27.88	0.0000
5	5A 5B	5.47 5.35	Y_{12}	10.82	5.41	117.07	0.0084
6	6A 6B	6.36 6.28	Y_{13}	12.64	6.32	159.77	0.0030
7	7A 7B	8.16 8.20	Y_{14}	16.36	8.18	267.65	0.0006
8	8A 8B	3.88 3.91	Y_{23}	7.79	3.90	60.68	0.0025
9	9A 9B	5.47 5.46	Y_{24}	10.93	5.47	119.46	0.0025
10	10A 10B	4.61 4.46	Y_{34}	9.07	4.54	82.27	0.0087
Control							
11	11A 11B	10.90 10.88	C_1	21.78	10.89	474.37	-0.0006
12	12A 12B	6.80 6.86	C_2	12.86	6.43	165.38	0.0956
13	13A 13B	2.88 3.12	C_3	6.00	3.00	36.00	0.0288
14	14A 14B	3.00 3.01	C_4	6.01	3.01	36.12	0.0001
15	15A 15B	0.62 0.60	C_5	1.22	0.61	1.49	-0.0006
16	16A 16B	3.98 4.10	C_6	8.08	4.04	65.29	-0.0054
17	17A 17B	0.50 0.52	C_7	1.02	0.51	1.04	0.0004
18	18A 18B	15.36 15.04	C_8	30.40	15.20	924.16	0.0512
19	19A 19B	6.01 6.00	C_9	12.01	6.01	144.24	0.0001
20	20A 20B	12.86 12.82	C_{10}	25.68	12.84	659.46	0.0020
						Σ	**0.2077**

4.3. The regression equation

The solution of Equation (16b), given the responses in Table 4, gives the unknown coefficients of the regression equation (Equation 14) as follows:

$$\beta_1 = 421987.5, \quad \beta_2 = 8238.0, \quad \beta_3 = -319.5,$$
$$\beta_4 = 261.7, \; \beta_{12} = -793265.6, \; \beta_{13} = -475248.6,$$
$$\beta_{14} = -488849.6, \; \beta_{23} = 35769.5, \; \beta_{24} = 44916.9,$$
$$\beta_{34} = 1244.5$$

Thus, from Equation (14) the regression is given by:

$$\hat{Y} = 421987.5Z_1 + 8238.0Z_2 - 319.5Z_3 + 261.7Z_4 - 793265.6Z_1Z_2 - 475248.6Z_1Z_3 - 488849.6Z_1Z_4 + 35769.5Z_2Z_3 + 44916.9Z_2Z_4 + 1244.5Z_3Z_4 \quad (17)$$

Equation (17) is the mathematical model for the optimization of shear modulus of the Rice Husk Ash Pozzolan concrete, based on Osadebe's second-degree polynomial.

4.4. Test of the adequacy of the model

Equation (17), the model equation, was subjected to statistical student's t- test and Fisher test for adequacy against the controlled experimental results. It was proved adequate. A typical result of an executed program is shown in appendix B.

5. Conclusion

The research showed that the Rice Husk Ash (RHA) produced an average value of shear modulus of 8.1N/mm^2 with an average Poisson ratio of 0.26.The model equation was tested for adequacy using the student's t-test and the Fisher test. The strengths predicted by the model are in good agreement with the corresponding experimentally obtained results. With the model, any desired strength of hardened concrete, given any mix proportions, is easily evaluated. Conversely, the various mix proportions matching any stipulated strength are also easily obtained using simple BASIC computer program. The output of an executed program is shown in Appendix A. The program is presented in Appendix B.

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Appendix A: An executed program for shear modulus (Osadebe's)

Desired strength? 5.0

Counter	z1	z2	z3	z4	y	s1	s2	s3	s4
1	0.080	0.098	0.551	0.271	4.999	0.859	1.000	2.040	3.453
2	0.085	0.105	0.536	0.274	5.000	0.859	1.000	2.043	3.458
3	0.085	0.119	0.643	0.153	5.000	0.858	1.000	2.043	3.525
4	0.098	0.112	0.316	0.474	5.000	0.860	1.000	2.049	3.368
5	0.098	0.220	0.356	0.326	5.000	0.860	1.000	2.049	3.496

Appendix B: Program for Osadebe's model

```
10 REM A GW BASIC V2.02 program that computes the proportions of concrete mixes to a desired strength.
20 REM Osadebe's model
30 COUNT = 0
40 GOSUB 100
50 END
100 REM procedure begins
110 PRINT  "A Model for Computation of Concrete Mix Proportions to a Desired Strength"
120 PRINT
130 INPUT "Desired Strength"; YIN
140 GOSUB 400
150 FOR Z1 = 0.08 TO 0.1 STEP .001
160 FOR Z2 = 0.09 TO 1-Z1 STEP .001
170 FOR Z3 = 0.2 TO 1-Z1-Z2 STEP .001
180 Z4 = 1-Z1-Z2-Z3
190 REM Assign Coefficients
200 B1 =
210 B2 =
220 B3 =
2230 B4 =
240 B12 =
250 B13 =
260 B14 =
270 B23 =
280 B24 =
290 B34 =
300 YOUT= B1*Z1+B2*Z2+B3*Z3+B4*Z4+B12*Z1*Z2+B13*Z1*Z3+B14*Z1*Z4+B23*Z2*Z3+B24
*Z2*Z4+B34*Z3*Z4
```

```
310 IF (ABS (YIN-YOUT)<=0.001) THEN 320 ELSE 340
320 COUNT = COUNT+1
330 GOSUB 500
340 NEXT Z3
350 NEXT Z2
360 NEXT Z1
370 RETURN
400 REM print heading
410 PRINT
420 PRINT "COUNT   Z1    Z2    Z3    Z4    Y    S1    S2    S3    S4"
430 RETURN
500 REM Outresults
510 S1 = 0.88*Z1+0.86*Z2+0.85*Z3+0,84*Z4
520 S2 = Z1+Z2+Z3+Z4
530 S3 = 2.8*Z1+2.0*Z2+2.5*Z3+2.2*Z4
540 S4 = 4.5*Z1+4.0*Z2+3.5*Z3+3.0*Z4
550 PRINT TAB (1); COUNT; USING "####.##";Z1;Z2;Z3;Z4;YOUT;S1;S2;S3;S4
560 RETURN
```