

EPQ model with depreciation cost and process quality cost as continuous functions of time

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Abstract

Extensive research has been devoted to economic production quantity (EPQ) problem. However, little attention has been paid to problems where depreciation cost and process quality cost must be considered, simultaneously. In this paper, we consider the economic production quantity model of minimizing the annual total cost subject to depreciation cost and process quality cost, where depreciation cost and process quality cost are assumed to be continuous functions of holding time and of production run length, respectively. Local search meta-heuristics: iterated local search (ILS) and simulated annealing (SA) are proposed to solve proposed model. Finally, the meta-heuristics are computationally compared by using some numerical examples and results are analyzed.

Keywords: EPQ; Inventory control; Process quality; Depreciation cost; Iterated local search; Simulated annealing

1. Introduction

The economic production quantity (EPQ) model has been widely used in practice because of its simplicity. However, there are some drawbacks in the assumption of the original EPQ model and many researchers have tried to improve it with different viewpoints, and the absence of the inventory quality is one of these shortcomings.

The classic EPQ model assumes that quality level is fixed at an optimal level. However, in real production environment, this assumption does not accurately reflect the reality. Because, it can often be observed that the product quality depends on the production run time of the production process and the holding time. Hence, the inventory policy determined by the conventional model might be inappropriate. The relationship between quality and EPQ model has been diversely studied over the last decade. Porteus initially studied the effect of process deterioration on the optimal production cycle time [13]. Rosenblatt and Lee conducted that the presence of defective products motivates smaller lot sizes [14]. Tapiero links optimal

quality inspection policies and the resulting improvements in the manufacturing cost [16]. Lee and Rosenblatt considered using process inspection during the production run so that the shift to out-of-control state can be detected and restored earlier [10]. Kim and Hong [7] extended the work of Lee and Rosenblatt [10] by assuming that an elapsed time until shift is arbitrarily distributed.

Chan et al. modified the classical EPQ model in order to develop a new EPQ model that can be accommodated with different situations and taking consideration of the time factor [1]. Papachristos and Konstantaras discussed the issue of non-shortages in inventory models with imperfect quality and especially in models with proportional imperfect quality, i.e. models where the rate of defectives is a random variable [12].

Khouja reformulated some inventory models which allow for adjustments to the process within a production cycle, without interrupting the system [6]. These adjustments (minor setups) do not involve performing all activities of a full set up and incurs only a fraction of a full set up cost and time. Jaber [4] extended the

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work of Khouja [6] by assuming that the setup cost reduces because of learning effects, and that the rate of generating defects reduces because the production process benefits from any changes for eliminating the defects, and thus reduces with every minor setup.

Salameh and Jaber considered a special inventory situation where items, received or produced, are not of perfect quality [15]. Hou and Lin studied the effect of an imperfect production process on the optimal production run length when capital investment in process quality improvement is adopted [3]. The optimal lot sizing and capital investment are appropriately determined, and relevant critical performance measures are also discussed. Lee presented a cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system in order to increase product and service quality [9].

Recently, Hou considered an EPQ model with imperfect production processes, in which the setup cost and process quality are functions of capital expenditure [2]. Tsou presented a modified inventory model which accounts for imperfect items and Taguchi's cost of poor quality [19].

Tsou and Chen developed a quality improvement model based on the hypothesis of a classical EPQ model [20]. The practical case of a car seat assembly line was used to verify this model. Tsou and Chen proposed a dynamic model for a defective production system with Poka Yoke [21].

The practical case of automotive industry was used to verify proposed model. Tsou formulated a model on quality investment in a dynamic lot sizing production system [18].

None of the above surveyed works assumed depreciation cost and process quality cost to be continuous functions of holding time and of production run length, respectively. The rationales for these assumptions are as follows:

In the beginning of production cycle, produced items have lower quality due to instability of production equipment and learning. After some time, quality of produced items has the best situation, because of stability of production system and learning. Again at end of production long cycle, produced items have lower quality due to fatigue of operators and equipments, necessity to service operations etc. (see Jaber and Bonney [5] and Urban [22] about influential learning and forgetting in quality of produced items). In this paper cost due to the influence of length production cycle in reduction quality, named quality cost.

Produced items in warehouses will be deteriorated and generally, in reality the value or utility of goods,

while in stock, may decrease. Then the value of produced items may be depending on the holding time. The relation between depreciation cost and holding time may not be linear. In this paper cost imposed because of the influence of holding time in reduction quality named depreciation cost that is similar to depreciation cost in financial models.

These assumptions add complexity of the model where a closed form solution was not possible and the convexity of the cost function was not validated. This therefore requires relying on meta-heuristics search methods such as iterated local search (ILS) and simulated annealing (SA) to solve this problem.

In this paper, we use ILS and present a method to solve the economic production quantity model of minimizing the total annual cost subject to depreciation cost and process quality, where depreciation cost and process quality costs are assumed to be continuous functions of holding time and of production run length, respectively. The paper is organized as follows:

Section 2 describes the problem. Section 3 explains the basics of simulated annealing (SA) and iterated local search (ILS). In Section 4, we explain the steps of SA and ILS algorithm to solve the problem. Some numerical examples and its computational results are represented in Section 5. Finally, Section 6 contains the conclusions.

2. Problem description

In this section, we derive a mathematical statement for the EPQ model with depreciation cost and process quality cost. The basic EPQ model is that of determining a production quantity of an item, subject to the following conditions related to the production facility and marketplace [17]:

- Demand rate and production rate are continuous, known and constant. Production rate is greater than or equal to demand rate.
- All demand must be met.
- Holding costs are determined by the value of the item.
- Setup time is assumed to zero.
- Unit production cost of product and setup cost are time and quantity invariant.
- There are no quantity constraints.
- No shortages are allowed.

Most of the assumptions in our mathematical model are the same as those in the conventional EPQ. Besides, we impose the following additional assumptions:

- 1) Product depreciation depends on its holding time.
- 2) The depreciation cost of product is a non-decreasing function of its holding time.
- 3) The process quality cost of product is a convex function of run time.

The first assumption is very clear because in financial models depreciation depend on time, then after producing item it may have depreciation by time (holding time).

The second assumption is also related to concept of deprecation because generally deprecation cannot be decreasing function.

The third assumption is very realist. Quality in the beginning of the production cycle is low due to non learning operators and instability of production equipments. After some time, quality of product will be in the best situation. Again, by passing time the quality of products is going to low due to fatigue of the operators and equipments, necessity to service operations etc. That means the process quality is a concave function of production run length. Therefore, we assume the process quality cost of product is a convex function of run time.

In classic EPQ model value or quality of goods are assumed to be independent of holding time and production run length. Generally, in reality the value or utility of goods, while in stock, may decrease in case of deteriorating items. Stored items may be unprofitable depending on holding time. We suppose that depreciation cost is a continuous non-decreasing function $h(t)$ of holding time. Also, production run length can influence the quality of produced goods. As explanation at above, we suppose that process quality cost is a continuous convex function $g(t)$ of production run length.

In order to state the problem mathematically, let:

- Q Production quantity (real positive decision variable).
- Q'' Local optimum production quantity.
- Q^* Economic production quantity (positive real decision).

- D Annual demand rate of product.
- P Annual production rate of product.
- C Unit production cost of product.
- h Annual unit holding cost.
- A Fixed setup cost of production system.
- T Cycle length.
- T_p Production period length in a cycle.
- T_d Only-demand period length in a cycle.
- $h(t)$ Non-decreasing continuous function of depreciation cost.
- $g(t)$ Convex continuous function of process quality cost.
- $I_p(t)$ Inventory level at time t of production period.
- $I_d(t)$ Inventory level at time t of only-demand period.
- ATC Annual total cost (objective function).

The behavior of inventory level in EPQ model is illustrated in Figure1.

The objective is to find economic production quantity Q^* , in order to minimize the annual total cost ATC. ATC is computed as follows:

$$ATC = CD + \frac{D}{Q}A + \frac{h}{2}Q\left(1 - \frac{D}{P}\right) + \frac{D}{Q} \int_0^{T_p} P g(t)dt + \frac{D}{Q} \int_0^{T_p} h(t)I_p(t)dt + \frac{D}{Q} \int_0^{T_d} h(t)I_d(t)dt \quad (1)$$

where the first term is production cost. The second term is setup cost. The third term is the holding cost that related to investment cost and costs of warehousing and it dose not contain the cost of the deterioration products. The forth term is the process quality cost. Fifth and sixth terms are depreciation cost of inventory holding that is different from the third term and $h(t)$ may have any non-decreasing function.

From graphical representation of EPQ model in Figure 1, we have:

$$T_p = \frac{Q}{P} \quad (2)$$

$$T_d = \frac{Q}{D} - \frac{Q}{P} \quad (3)$$

$$I_p(t) = (P - D)t \quad (4)$$

$$I_d(t) = I_{\max} - Dt \quad (5)$$

Substituting Equations (2), (3), (4) and (5) into Equation (1) yields:

$$\begin{aligned} ATC = & CD + \frac{D}{Q}A + \frac{h}{2}Q\left(1 - \frac{D}{P}\right) + \frac{D}{Q}\int_0^{\frac{Q}{P}} P g(t) dt \\ & + \frac{D}{Q}\int_0^{\frac{Q}{P}} h(t)(P - D)t dt \\ & + \frac{D}{Q}\int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} h(t)(I_{\max} - Dt) dt \end{aligned} \quad (6)$$

Also, from graphical representation of EPQ model in Figure 1, we have:

$$I_{\max} = Q\left(1 - \frac{D}{P}\right) \quad (7)$$

Substituting Equation (7) into Equation (6) yields the following expression of the annual total cost ATC:

$$\begin{aligned} ATC = & CD + \frac{D}{Q}A + \frac{h}{2}Q\left(1 - \frac{D}{P}\right) + \frac{PD}{Q}\int_0^{\frac{Q}{P}} g(t) dt \\ & + \frac{(P - D)D}{Q}\int_0^{\frac{Q}{P}} th(t) dt + D\left(1 - \frac{D}{P}\right)\int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} h(t) dt \\ & - \frac{D^2}{Q}\int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} th(t) dt \end{aligned} \quad (8)$$

The first order condition for Q yields its optimal choice (see appendix A):

$$\frac{d}{dQ} ATC = \frac{h}{2}\left(1 - \frac{D}{P}\right) - \frac{DA}{Q^2} - \frac{PD}{Q^2}\int_0^{\frac{Q}{P}} g(t) dt$$

$$\begin{aligned} & + \frac{Q}{D}g\left(\frac{Q}{P}\right) - \frac{(P - D)D}{Q^2}\int_0^{\frac{Q}{P}} th(t) dt \\ & + \frac{D(P - D)}{P^2}h\left(\frac{Q}{P}\right) + \frac{D^2}{Q^2}\int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} th(t) dt \end{aligned} \quad (9)$$

Notice that the ATC in Equation (8) is nor convex or concave (since it's second derivative is sign free (see Appendix B)). Then Q'' is a local minimum solution of ATC if:

$$\frac{d}{dQ} ATC(Q'') = 0 \quad (10)$$

and

$$\frac{d^2}{dQ^2} ATC(Q'') > 0 \quad (11)$$

Replacing Equation (9) in Equation (10) yields:

$$\begin{aligned} & D\int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} th(t) dt - (P - D)\int_0^{\frac{Q}{P}} th(t) dt \\ & - P\int_0^{\frac{Q}{P}} g(t) dt + \frac{(P - D)}{P^2}Q^2h\left(\frac{Q}{P}\right) + Qg\left(\frac{Q}{P}\right) \\ & + \frac{h}{2D}Q^2\left(1 - \frac{D}{P}\right) = A \end{aligned} \quad (12)$$

This problem poses a difficult computational task due to the nonlinearities involved, depending on functions $h(t)$ and $g(t)$. Due the complexity to solve this equation, we developed a method based on iterated local search (ILS) metaheuristic.

3. General simulated annealing and iterated local search

3.1. Simulated annealing

Simulated annealing is one of the most novel algorithms initially presented by Kirkpatrick et. al. [8]. Similar to other meta-heuristic algorithms, such as Genetic algorithm, Tabu search, and Ant algorithm, it attempts to solve hard combinatorial optimization problems through controlled randomization. Ease of

use and provision of good solutions to real-world problems makes this algorithm be one of the most powerful and popular meta-heuristics to solve many optimization problems.

The basic structure of SA algorithm is presented in Table 1, where the following notation is used:

S	The current solution.
S^*	The best solution.
S_n	Neighboring solution.
$f(S)$	The value of objective function at solution S .
n	Repetition counter.
T_0	Initial temperature.
L	Number of repetition allowed at each temperature level.
p	Probability of accepting S_n when it is not better than S

It is obvious that this procedure just takes into account the minimization problems, hence while performing a maximization problem, the objective function is multiplied by (-1) to obtain a capable form.

The algorithm starts with an initial solution for the problem. As it is obvious from table 1, SA has two cycles, inner and outer. In the inner cycle of the SA, repeated while $n < L$, a neighboring solution S_n of the current solution S is generated.

If $\Delta \leq 0$ (S_n is better than S), then the generated solution replaces the current solution, otherwise the solution is accepted with a criterion probability, say $p = e^{-\Delta/T}$. The value of the temperature, T , decreases in each iteration of the outer cycle of the algorithm.

As a meta-heuristic algorithm, the most important feature of this algorithm is the possibility of accepting a worse solution, hence allowing it to prevent falling into a local optimum trap. Obviously, the probability of accepting a worse solution decreases as the temperature decreases in each outer cycle. The performance of SA depends on the definition of the several control parameters:

1. The initial temperature (T_0) should be high enough that in the first iteration of the algorithm, the probability of accepting a worse solution is, at least, of 80% [2].

2. The most commonly used temperature reducing function is geometric; i.e. $T_i = CT_{i-1}$ in which $C < 1$ and constant. Typically, $0.75 \leq C \leq 0.95$.

3. The length of each temperature level (L) determines the number of solutions generated at a certain temperature, T .

4. The stopping criterion defines when the system has reached a desired energy level. Equivalently it defines:

- The total number of solutions generated,
- The temperature at which the desired energy level is reached (freezing temperature),
- The acceptance ratio (ratio between the number of solutions accepted and the number of solutions generated).

It is obvious that these control parameters are chosen with respect to the specific problem at hand. When adapting this general algorithm to a specific problem, the procedure to generate both initial and neighboring solutions is very important in addition to the control parameter.

3.2. Iterated local search

Iterated local search is a simple but powerful meta-heuristic algorithm [11]. It applies local search to an initial solution until it finds a local optimum; then it perturbs the solution and it restarts local search. The importance of the perturbation is obvious: too small a perturbation might not enable the system to escape from the basin of attraction of the local optimum just found. On the other side, too strong a perturbation would make the algorithm similar to a random restart local search.

A local search is effective if it is able to find good local optima, that is, if it can find the basin of attraction of those states. When the search space is wide and / or when the basin of attraction of good local optima is small, a simple multi-start algorithm is almost useless. An effective search could be designed as a trajectory only in the set of local optima s^* , instead of in the set s of all the states. Unfortunately, in most cases there is no feasible way of introducing a neighborhood structure for s^* . Therefore, a trajectory along local optima $s_1^*, s_2^*, \dots, s_t^*$ is performed, by applying the following scheme:

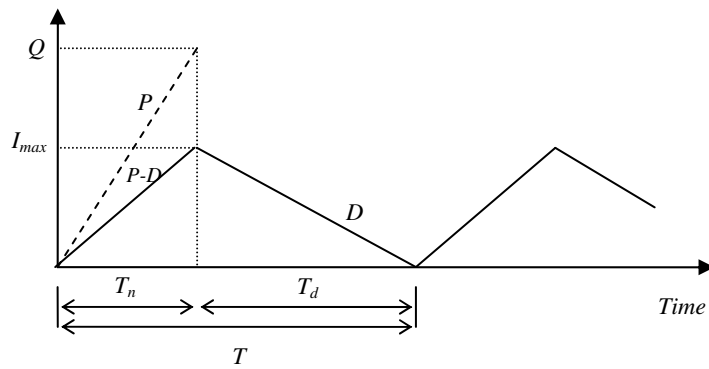


Figure 1. The relation between inventory level and time.

Table 1. Simulated Annealing Algorithm for minimization problem.

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Initialize the SA control parameter ( $T_0, L$ )
Select an initial solution,  $S_0$ 
Set  $T=T_0$ ; Set  $S=S_0$ ; Set  $S^*=S_0$ ; Calculate  $f(S_0)$ ;
While the stop criterion is not reached do:
  Set  $n = 1$ ;
  While  $n < L$  do:
    Generate solution  $S_n$  in the neighborhood of  $S_0$ ; Calculate  $\Delta = f(S_n) - f(S)$ ;
    if  $\Delta \leq 0$ 
       $S=S_n$ 
    else
      generate a random number,  $r \in (0, 1)$ 
      if ( $r \leq p = e^{-\Delta/T}$ );
         $S=S_n$ ;  $n = n+1$ ;
      end
    end
  end
  if ( $f(S) < f(S^*)$ )
     $S^* = S_n$ ;
  end
end
reduce the temperature  $T$ ;
end

```

- 1) Execute local search from an initial state s until a local optimum s^* is found.
- 2) Perturb s^* and obtain s' .
- 3) Execute local search from s' until a local optimum s'^* is reached.
- 4) On the basis of an *acceptance criterion* decide whether to set $s^* \leftarrow s'$.
- 5) Go to step 2.

The requirement on the perturbation of s is to produce a starting point for local search such that a local optimum different from s is reached. However, this new local optimum should be closer to s than a local optimum produced by a random restart. The acceptance criterion acts as a counter balance, as it filters and gives feedback to the perturbation action, depending on the characteristics of the new local optimum. A high level description of ILS steps is presented in Table 2.

The design of ILS algorithms has several degrees of freedom in the choice of the initial solution, perturbation and acceptance criteria.

The construction of initial solutions should be fast, and initial solutions should be a good starting point for local search. The fastest way of producing an initial solution is to generate it at random; however, this procedure being the easiest way to produce a solution for unconstrained optimization problems should be carefully adjusted to generate feasible solutions for constrained problems. The perturbation is usually nondeterministic in order to avoid cycling. Its most important characteristic is the strength, simply defined as the amount of changes made on the current solution. The strength can be either fixed or variable. In the first case, the distance between perturbation and Local Search is kept constant, independently of the problem size. However, a variable strength is in general more effective, since it has been experimentally found that, in most of the problems, the bigger the problem size, the larger should the strength be.

A second choice is the mechanism to perform perturbations. This may be a random mechanism, or the perturbation may be produced by a deterministic or semi-deterministic method.

The third important component is the acceptance criterion. Two extreme examples can be defined as (1) accepting the new local optimum only in case of improvement and (2) always accepting the new solution. In-between, there are several possibilities. For

example, it is possible to adopt a kind of annealing schedule: accept all the improving new local optima and accept also the non-improving ones with a probability that is a function of the temperature τ and the difference of objective values, in formulas:

$$P(\text{Acceptance}) = \begin{cases} 1 & \text{if } f(s'^*) < f(s^*) \\ \exp\left(-\frac{f(s'^*) - f(s^*)}{\tau}\right) & \text{otherwise} \end{cases} \quad (13)$$

4. Using SA and ILS in proposed model

In this section we proposed two metaheuristic methods to solve the proposed model.

4.1. Applying ILS algorithm to the problem

To obtain the economic production quantity of the above mentioned model we are to minimize ATC, using iterated local search. In this regard, the steps of this algorithm are briefly presented bellow where the following notation is used:

- Q_0 Initial solution.
- Q Current solution.
- Q'^* Local optimum solution.
- Q^* Best solution.
- $ATC(Q)$ Value of the objective function at solution Q .
- ϵ Neighborhood step-length parameter.
- λ Perturbation step-length parameter.
- k Repetition counter.
- L Number of repetitions allowed.

Initialize the ILS control parameter (λ, ϵ, L)

select an initial solution Q_0 between 0 and D ;

set $Q = Q_0$;

while $|ATC'(Q)| > \epsilon$ do

if $ATC'(Q) < 0$

$Q = Q + \epsilon$;

else

```

    if  $ATC'(Q) > 0$ 
         $Q = Q - \varepsilon$ ;
    end
end
set  $Q^{*} = Q$ ;
set  $Q^{*} = Q^{*}$ ;
set  $k = 1$ ;
while  $k < L$  do:
    generate a binary random number  $z \in [0,1]$ 
     $Q = z \cdot [Q^{*} + \lambda \cdot (D - Q^{*})] + (1-z) \cdot \lambda \cdot Q^{*}$ ;
    while  $|ATC'(Q)| > \varepsilon$  do
        if  $ATC'(Q) < 0$ 
             $Q = Q + \varepsilon$ ;
        else
            if  $ATC'(Q) > 0$ 
                 $Q = Q - \varepsilon$ ;
            end
        end
    end
    set  $Q^{*} = Q$ ;
    if  $ATC(Q^{*}) \leq ATC(Q^{*})$ 
        set  $Q^{*} = Q^{*}$ ;
    end
     $k = k + 1$ ;
    reduce the Perturbation step-length parameter  $\lambda$ ;
end.

```

The algorithm starts with an initial solution (production quantity) for the problem between 0 and D , at random, and by initializing the so-called perturbation step-length parameter λ , the neighborhood step-length parameter ε and the number of repetitions allowed, L .

To generate neighborhood solutions we use Equation (12): If the first derivative of Equation (8) at solution Q is negative, a neighborhood solution is generated by right shift of current solution by an amount of ε , and if the first derivative of Equation (8) at solution Q is positive, a neighborhood solution is generated by left shift of current solution by an amount of ε , and then the generated solution replaces the current one. This procedure continues until a local optimum solution Q^{*} is reached, namely, the first derivative of Equation (8) at solution Q almost equals to zero. This first local optimum solution sets as best

solution Q^{*} . In the inner cycle of ILS, repeated while $k < L$, a perturbed solution of the current local optimum solution Q^{*} is generated as follows: with generating a binary random number z , we select a direction for perturbation (right or left).

Perturbed solution is obtained by adding to or subtracting from current local optimum solution Q^{*} , a dynamic amount, depending on the perturbation direction and λ , where $\lambda \in [0,1]$ is the perturbation step-length parameter playing an important role in our algorithm. The generated solution replaces the current one. Local search procedure is applied to the newly chosen solution. We suppose that after the local optimum is reached, it is always acceptable. Hereby, the most important feature of this algorithm, as a metaheuristic, is the possibility of accepting a worse solution, which can allow it to prevent falling into local optimum trap.

The choice of an appropriate λ is crucial for the performance of the algorithm. The value of parameter λ decreases during the search process, thus at the beginning of the search, diversification is high and as it gradually goes on its search path, intensification becomes more apparent. The terms diversification generally refers to the exploration of the search space, whereas the term intensification refers to the exploitation of the accumulated search experience. Hereby, with the choice of an appropriate λ , a dynamic compromise is made between diversification and intensification.

In proposed model the values of Q must be integer. This can be problematic, because it may be the case that the optimum be an integer neighborhood of a local but not the global solution.

To overcome this problem in this algorithm we save the immediate integer neighborhoods of each solution in each iteration to make sure that we pick the right solution.

4.2. Applying SA algorithm to the problem

For SA the initial solution is chosen to be an amount equal to the half of the demand. Integer neighborhoods are generated by adding or subtracting an integer value, say neighborhood length, to or from the current solution. The neighborhood length depends on temperature. We set $L = 5$ and $C = 0.95$; but we set T_0 and T_f at different values for different problem sizes.

Table 2. Iterated Local Search (ILS) Algorithm.

$s_0 \leftarrow$ Generate Initial Solution ()
 $s^* \leftarrow$ Apply Local Search(s_0)
 While termination condition not met do
 $s' \leftarrow$ Apply perturbation (s^*)
 $s'^* \leftarrow$ Apply Local Search (s')
 $s^* \leftarrow$ Apply acceptance criterion (s^*, s'^*)
 Memorize Best Found Solution
 End While

Table 3. Comparison SA, ILS and enumeration methods (P=1500, D=1300).

Index→	Q^*	ATC	Time (Sec)
Method			
↓			
Enumeration	1075	65974	469.49
Iterated Local Search	1075	65974	224.84
Simulated Annealing	1075	65974	182.43

Table 4. Comparison SA, ILS and Enumeration methods (P=2200, D=1900).

Index→	Q^*	ATC	Time (Sec)
Method			
↓			
Enumeration	1576	65974	682.24
Iterated Local Search	1576	65974	309.12
Simulated Annealing	1576	65974	180.09

Table 5. Comparison SA, ILS and enumeration methods (P=3500, D=3000).

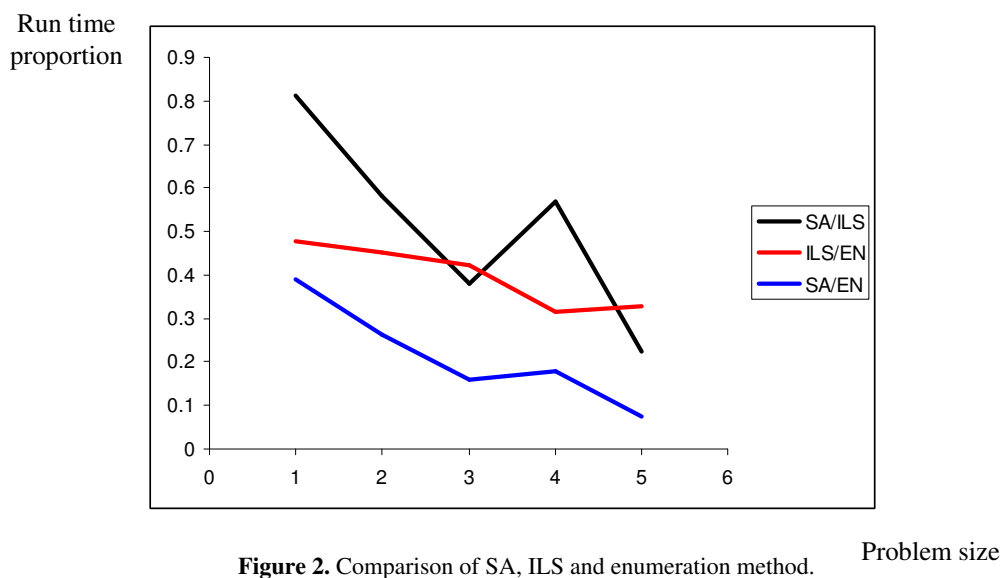
Index→	Q^*	ATC	Time (Sec)
Method			
↓			
Enumeration	2507	151990	1073.80
Iterated Local Search	2507	151990	453.93
Simulated Annealing	2507	151990	172.46

Table 6. Comparison SA, ILS and Enumeration methods (P=10000, D=14000).

Index→	Q^*	ATC	$Time$ (Sec)
Method ↓			
Enumeration	9998	509710	3332.6
Iterated Local Search	9998	509710	1046.40
Simulated Annealing	9998	509710	594

Table 7. Comparison SA, ILS and Enumeration methods (P=100000, D=130000).

Index→	Q^*	ATC	$Time$ (Sec)
Method ↓			
Enumeration	13	1838500	34069
Iterated Local Search	92955	5079500	11143
Simulated Annealing	92955	5079500	2503.50

**Figure 2.** Comparison of SA, ILS and enumeration method.

5. Numerical examples

In this section, we solve a problem instance for illustrative and comparative purposes. In addition, to know the effects of the perturbation rule and acceptance criterion, different approaches have been chosen and used to optimize Equation (8). Let us consider an inventory system with the following data:

$P=1500, 2200, 3500, 14000, 13000$ units/year,

$D=1300, 1900, 3000, 10000, 100000$ units/year,

$A=200, C=3, h=2, g(t) = e^{7(t-0.2)} - 30 \ln(t),$

$h(t) = (C - C/5)e^{0.005t}.$

We select $\lambda = 5$ in ILS.

The initial solution is considered as the half of the demand. Tables 3 to 7 show the different measure one can use to compare the performance of these algorithms. To be able to compare the algorithms in terms of timing and the minimum value that they achieve we also did a direct enumeration search to establish the model, but good to serve as a comparison platform for which the results are shown in corresponding tables.

Almost in all of the cases all the algorithms report the same EPQ, hence same objective function, therefore the only criterion that is left for the comparison sake is the run time. As obviously can be viewed from Figure 2, simulated annealing algorithm outperforms the other algorithms. Also, as the problem size gets bigger SA seems to perform much more efficiently.

Comparison of CPU time requirements of SA, ILS and enumeration method is shown in Figure 2. We could clearly see that for each dimension of model's parameters P and D , the simulated annealing and iterated local search have good performance in time by comparing with enumeration method, whereas the growth of the parameters dimension improves the efficiency of the simulated annealing algorithm respect to iterated local search.

6. Conclusion

In reality the value or utility of goods, while in stock, may decrease in case of deteriorating items. Also, production run length can influence the quality of produced goods. In this paper, economic production quantity (EPQ) model has been developed considering both the depreciation cost of stored items and process quality cost. We have assumed

depreciation cost to be a continuous non-decreasing function of holding time, and process quality cost to be a continuous convex function of production run length. The problem has been described using a mathematical model, and then the simulated annealing (SA) and Iterated Local Search (ILS) have been proposed to solve it. From the numerical results, we clearly see that the ILS algorithm performs better than the enumeration method. On the other hand, SA outperforms ILS.

References

- [1] Chan, W. M., Ibrahim, R. N. and Lochert, P. B., 2003, A new EPQ model: integrating lower pricing, rework and reject situations. *Production Planning and Control*, 14(7), 588-595.
- [2] Hou, K. L., 2007, An EPQ model with setup cost and process quality as functions of capital expenditure. *Applied Mathematical Modeling*, 31, 10-17.
- [3] Hou, K. L. and Lin, L. C., 2004, Optimal production run length and capital investment with an imperfect production processes. *International Journal of System Sciences*, 35, 133-137.
- [4] Jaber, M. Y., 2006, Lot sizing for an imperfect production process with quality corrective interruptions and improvements, and reduction in setups. *Computers and Industrial Engineering*, 51(4), 781-790.
- [5] Jaber, M. Y. and Bonney, M., 2003, Lot sizing with learning and forgetting in set-ups and in product quality. *International Journal of Production Economics*, 83(1), 95-111.
- [6] Khouja, M., 2005, The use of minor setups within production cycles to improve product quality and yield. *International Transactions in Operational Research*. 12(4), 403-416.
- [7] Kim, C. H. and Hong, Y., 1999, An optimal production run length in deteriorating production processes. *International Journal of Production Economics*, 58, 183-189.
- [8] Kirkpatrick, S., Gelatt, C. and Vecchi, M., 1983, Optimization by simulated annealing.
- [9] Lee, H. H., 2005, A cost / benefit model for investments in inventory and preventive maintenance in an imperfect production system. *Computers and Industrial Engineering*, 48, 55-68.

- [10] Lee, H. L. and Rosenblat, M. J., 1987, Simultaneous determination of production cycles and inspection schedules in a production system. *Management Sciences*, 33, 1125-1137.
- [11] Lourenco, H. R., Martin, O. and Stutzle, T., 2001, *A Beginner's Introduction to Iterated Local Search*. Proceeding of MIC'2001 Metaheuristics International Conference, volume1, 1-6, Porto, Portugal.
- [12] Papachristos, S. and Konstantaras, I., 2006, Economic ordering quantity models for items with imperfect quality. *International Journal of Production Economics*, 100, 148-154.
- [13] Porteus, E. L., 1986, Optimal lot sizing process quality improvement and setup cost reduction. *Operations Research*, 34, 137-144.
- [14] Rosenblat, M. J. and Lee, H. L., 1986, Economic production cycles with imperfect production processes. *IIE Transactions*, 18, 48-55.
- [15] Salameh, M. K. and Jaber, M. Y., 2000, Economic production quantity model for items with imperfect quality. *International Journal of Production Economics*, 64, 59-64.
- [16] Tapiero, C., 1987, Production learning and quality control. *IIE Transactions*, 19, 362-370.
- [17] Tersine, J. R., 1988, *Principles of Inventory and Materials Management*. 3rd ed., New York: North-Holland.
- [18] Tsou, J. C., 2006, Joint decision making on lot sizing and quality investment in a dynamic lot sizing production system. *International Journal of Systems Science*, 37(7), 463-475.
- [19] Tsou, J. C., 2007, Economic order quantity model and Taguchi's cost of poor quality. *Applied Mathematical Modeling*, 31, 283-291.
- [20] Tsou, J. C. and Chen, J. M., 2005, Case study: Quality improvement model in a car seat assembly line. *Production Planning and Control*, 16(7), 681-690.
- [21] Tsou, J. C. and Chen, J. M., 2005, Dynamic model for a defective production system with poka-yoke. *Journal of the Operational Research Society*, 56(7), 799-803.
- [22] Urban, T. L., 1998, Analysis of production systems when run length influences product quality. *International Journal of Production Research*, 36(1), 3085-3094.

Appendix A

Computation procedure of the first derivative of ATC - Equation (9).

Recall Equation (8):

$$\begin{aligned}
 ATC = & CD + \frac{D}{Q}A + \frac{h}{2}Q\left(1 - \frac{D}{P}\right) + \frac{PD}{Q} \int_0^{\frac{Q}{P}} g(t)dt \\
 & + \frac{(P - D)D}{Q} \int_0^{\frac{Q}{P}} t h(t)dt \\
 & + D\left(1 - \frac{D}{P}\right) \int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} h(t)dt \\
 & - \frac{D^2}{Q} \int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} t h(t)dt
 \end{aligned}$$

$$\begin{aligned}
 \frac{d}{dQ} ATC = & 0 - \frac{DA}{Q^2} + \frac{h}{2}\left(1 - \frac{D}{P}\right) \\
 & - \frac{PD}{Q^2} \int_0^{\frac{Q}{P}} g(t)dt + \frac{PD}{Q} \left[\left(\frac{1}{P}\right)g\left(\frac{Q}{P}\right) - 0\right] \\
 & - \frac{D(P - D)}{Q^2} \int_0^{\frac{Q}{P}} t h(t)dt \\
 & + \frac{D(P - D)}{Q} \left[\left(\frac{1}{P}\right)\left(\frac{Q}{P}\right)h\left(\frac{Q}{P}\right) - 0\right] \\
 & + D\left(1 - \frac{D}{P}\right)\left(\frac{1}{D} - \frac{1}{P}\right)h\left(\frac{Q}{D} - \frac{Q}{P}\right) \\
 & + \frac{D^2}{Q^2} \int_0^{\left(\frac{Q}{D} - \frac{Q}{P}\right)} t h(t)dt \\
 & - \frac{D^2}{Q} \left(\frac{1}{D} - \frac{1}{P}\right)\left(\frac{Q}{D} - \frac{Q}{P}\right)h\left(\frac{Q}{D} - \frac{Q}{P}\right)
 \end{aligned}$$

Finally, one obtains Equation (9) as follows:

$$\begin{aligned}
 \frac{d}{dQ} ATC = & \frac{h}{2}\left(1 - \frac{D}{P}\right) - \frac{DA}{Q^2} \\
 & - \frac{PD}{Q^2} \int_0^{\frac{Q}{P}} g(t)dt + \frac{D}{Q} g\left(\frac{Q}{P}\right)
 \end{aligned}$$

$$\begin{aligned}
& - \frac{D(P-D)}{Q^2} \int_0^{\frac{Q}{P}} t h(t) dt \\
& + \frac{D(P-D)}{P^2} h\left(\frac{Q}{P}\right) \\
& + \frac{D^2}{Q^2} \int_0^{\left(\frac{Q}{D}-\frac{Q}{P}\right)} t h(t) dt
\end{aligned}$$

Appendix B

Computation procedure of the second derivative of ATC .

Recall Equation (9):

$$\begin{aligned}
\frac{d}{dQ} ATC &= \frac{h}{2} \left(1 - \frac{D}{P}\right) - \frac{DA}{Q^2} \\
& - \frac{PD}{Q^2} \int_0^{\frac{Q}{P}} g(t) dt + \frac{D}{Q} g\left(\frac{Q}{P}\right) \\
& - \frac{D(P-D)}{Q^2} \int_0^{\frac{Q}{P}} t h(t) dt
\end{aligned}$$

$$\begin{aligned}
\frac{d^2}{dQ^2} ATC &= 0 + \frac{2DA}{Q^3} + \frac{2PD}{Q^3} \int_0^{\frac{Q}{P}} g(t) dt \\
& - \frac{PD}{Q^2} \left[\left(\frac{1}{P}\right) g\left(\frac{Q}{P}\right) - 0 \right] - \frac{D}{Q^2} g\left(\frac{Q}{P}\right) \\
& + \frac{D}{Q} \left(\frac{1}{P}\right) g'\left(\frac{Q}{P}\right) + \frac{2D(P-D)}{Q^3} \int_0^{\frac{Q}{P}} t h(t) dt \\
& - \frac{D(P-D)}{Q^2} \left[\left(\frac{1}{P}\right) \left(\frac{Q}{P}\right) h\left(\frac{Q}{P}\right) - 0 \right] \\
& + \frac{D(P-D)}{P^2} \left(\frac{1}{P}\right) h'\left(\frac{Q}{P}\right) \\
& - \frac{2D^2}{Q^3} \int_0^{\left(\frac{Q}{D}-\frac{Q}{P}\right)} t h(t) dt \\
& + \frac{D^2}{Q^2} \left[\left(\frac{1}{D} - \frac{1}{P}\right) \left(\frac{Q}{D} - \frac{Q}{P}\right) h\left(\frac{Q}{D} - \frac{Q}{P}\right) - 0 \right]
\end{aligned}$$

$$\begin{aligned}
\frac{d^2}{dQ^2} ATC &= \frac{2DA}{Q^3} + \frac{2PD}{Q^3} \int_0^{\frac{Q}{P}} g(t) dt \\
& - \frac{2D}{Q^2} g\left(\frac{Q}{P}\right) + \frac{D}{PQ} g'\left(\frac{Q}{P}\right) \\
& + \frac{2D(P-D)}{Q^3} \int_0^{\frac{Q}{P}} t h(t) dt \\
& - \frac{D(P-D)}{P^2Q} h\left(\frac{Q}{P}\right) + \frac{D(P-D)}{P^3} h'\left(\frac{Q}{P}\right) \\
& - \frac{2D^2}{Q^3} \int_0^{\left(\frac{Q}{D}-\frac{Q}{P}\right)} t h(t) dt \\
& + \frac{1}{Q} \left[\left(1 - \frac{D}{P}\right)^2 h\left(\frac{Q}{D} - \frac{Q}{P}\right) \right]
\end{aligned}$$

It is obvious that second derivative of ATC is sign free.