Technical Note:

An opportunity cost maintenance scheduling framework for a fleet of ships: A case study

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Abstract

The conventional method towards deriving schedule for a fleet of ships to minimize cost alone has the shortcoming of not addressing the problem of operation revenue losses associated with delays during maintenance at ships dockyards. In this paper, a preventive maintenance schedule for a fleet of ships that incorporates opportunity cost is presented. The idea is to assign a penalty cost to all idle periods that the ships spend at the dockyard. A version of the scheduling problem was defined as a transportation model of minimizing maintenance costs. Fixed maintenance duration and dockyard capacity were the two constraints of the formulation. Relevant data from a shipping firm owing 8 ships and a dockyard in Lagos with a maintenance capacity of three ships per month were collected over a 24-month period. The maintenance cost function was then formulated with the parameters estimated and the transportation tableau set up. The considered eight ships arrived at the dockyard between the 1st and 20th month, and were expected to spend between 2 to 5 months for preventive maintenance. The optimal schedule of the cost function resulted in ships 1 to 8 being idle for 74 months. The results of the study showed that to reduce the cost and delays, decisions for scheduling preventive maintenance of a fleet of ships should be based on opportunity cost.

Keywords: Preventive maintenance scheduling; Maintenance cost; Opportunity cost; Fleet of ships scheduling

1. Introduction

Ship vessels are expensive but high revenue-yielding assets, which require highly competent personnel for its operations and maintenance [11,36]. Thus, in this decade of economic turbulence, there is a need for proper control and monitoring of the container port industry [4,24,29,31]. This control could be in the form of quality improvement [30], shipping policy improvement reformation [33], averting financial risks [16], improving productivity [10], benchmarking activities [3], and the introduction of opportunity cost concept [2,32]. Thus, in the decade of economic turbulence, there is a need for proper control and monitor of ship operational and maintenance activities. Unfortunately, in Nigeria, ship activity control in some firms seems to be weak with several days of idleness experienced by ships at the dockyard. Excuses to justify these huge revenue losses by ship waiting for service are not tenable. Since the operating funds are usually borrowed from banks, accumulation of high interests on lending is a challenge for

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company to develop a mechanism that could control ship delays for maintenance at dockyards. It then becomes necessary to introduce the concept of opportunity cost as a control mechanism to check unnecessary delay of ships at dockyards.

Opportunity cost relates to the revenue losses for the ships for being idle. This is equivalent to the market value of services that the ship would have rendered if not idle or being maintained. Several accounts of application of opportunity costs abound in the literature. Unfortunately, the case with ship maintenance and operations appear missing. With the incorporation of opportunity cost concept into the modeling of preventive maintenance scheduling problem, the shortcoming of the conventional method would be addressed [2,32]. This opportunity cost, when treated as a penalty for a team, would challenge the team members towards an improved performance at work. Consider the problem of absenteeism at ship dockyards where junior staffs arrive to work at will, and may be absent for some days without prior notice. Any staff that exhibits this unsatisfactorily may be fired since a huge penalty cost may have accrued to the team due to the tenancy. Thus, the ship maintenance manager need not be around all the time to monitor this team’s performance, but only need to refer to records.

Several investigations have been carried out on the development of systematic approaches to the solution of maintenance scheduling problems [17,19,21,22]. An interesting theme of research on maintenance scheduling relates to experimentation and modeling with uncertainties [12,14,26,27,28]. However, despite the wide scholarly activities on fuzzy-based maintenance scheduling, scholars have unconsciously omitted the incorporation of opportunity cost into modeling frameworks. The application of genetic algorithm to maintenance scheduling is also well-documented in print. The central theme of research is the development of genetic programming approaches that would optimize cost, time, human and non-human resources in the organizations [9,25,34,35,37]. Unfortunately, this growing theme of research seems to have ignored the concept of opportunity cost which would contribute to optimization of resources in organizations.

A number of successful research explorations have been documented on the fusion of artificial intelligence tools [8,17,18]. Again, the omission of opportunity cost concept is noticed in these evolving studies on hybrid genetic-fuzzy methodologies. Several other accounts on different aspects of maintenance scheduling have been studied in interesting areas such as aircraft [5,7,13,15], railway systems [20,23] and irrigation systems [1]. Also, there seems to be an omission of opportunity cost concept in these studies.

Having considered some of these important studies in areas of genetic algorithms, fuzzy systems, hybrid fuzzy and genetic systems and general literature on maintenance modeling and application, it is safe to conclude that the concept of opportunity cost has been ignored in the maintenance scheduling literature. Bearing in mind the economic significance and the possible human efficiency utilization implication of modeling maintenance scheduling with the use of opportunity cost, this work is recommended to bridge this important gap.

The structure of the paper is as follows: Introduction, mathematical framework, case study, results, discussion and conclusion. The introduction provides the motivation for the study and a justification from the literature of the need to close the wide gap that the absence of opportunity cost has caused. Section 2 discusses the mathematical model formulation of the problem and development of a solution procedure.

In Section 3, the case investigation of a shipping firm is considered to validate the model discussed in Section 2. This is followed by a summary of results in Section 4. Discussion of results is given in Section 5. This explains the results obtained from the study and their importance. Section 6 provides the concluding remarks for the study.

2. Mathematical framework

The mathematical model formulated for the problem is basically hinged on the concept of hybrid transportation solution technique called tradvar in view of the contiguous nature of allocation of the unit cost of resources. Maintenance schedules are allocated one after the other on the assumption that all resources needed to implement the maintenance task are available. In deriving the model, there is a need to observe the relationship between this and the previous model. This relationship is in terms of the components of $c_{ij}$. The strong feature of this model is to track the opportunity cost which is defined as loss in productive value that the facility could potentially generate. It should be noted that if facilities have to be withdrawn from operation for maintenance, then it must be for a short period to minimize the opportunity cost. However, the schedulers must control the quality of service since poor service could lead to machine performance degradation, which would ultimately translate to higher running costs and breakdown that could eventually offset whatever savings
were made in using few maintenance periods. This asserts the assumption that maintenance periods are known.

Reference to Charles-Owaba [6], \( c_{ij} = a_i + d_i (j - k_i) \) is stated, where \( a_i \) and \( d_i \) are referred to as cost parameters, and \( k_i \) is referred to as the arrival periods of facilities. The unit cost, \( c_{ij} \) becomes \( a_i \) if there is non-avoidable delays in the system. In this case, \( j = k_i \). That is, as the facilities arrive for maintenance, they are immediately serviced, and released for normal operational activities. However, if when facilities arrive for maintenance, and they are expected to spend time, \( t_i \) in the system, if this \( t_i \) is less than \( j - k_i \), the excess time is either used in operations (facilities are released for operation) or in maintenance. This excess time creates the idle period i.e. \([ (j - k_i) - t_i ]\). Thus, the idle cost is related to \( d_i [ (j - k_i) - t_i ] \). Thus, \( c_{ij} \) is redefined as \( c_{ij} = a_i + d_i [ (j - k_i) - t_i ] \). Thus,

\[
z = \sum_{i=1}^{M} \sum_{j=1}^{T} [a_i + d_i [ (j - k_i) - t_i ]] y_{ij}.
\]

The linear program could be established for the model as:

Minimize \( z = \sum_{i=1}^{M} \sum_{j=1}^{T} [a_i + d_i [ (j - k_i) - t_i ]] y_{ij} \)

Subject to:

\[
\sum_{j=1}^{T} y_{ij} = \sum_{i=1}^{N} B_i ,
\]

\[
\sum_{i=1}^{M} y_{ij} \leq A_j ,
\]

\( y_{ij} \geq 0 \).

Here, \( y_{ij} \) is a binary Gantt charting variable that is assigned a value of 1 when maintenance activities are carried on a ship, and a value of zero when the ship is either in operation or idle. The notation \( a_i \) represents the cost parameter with which optimization is to be done. The notation \( A_j \) is simply the maintenance capacity in period \( j \). This means the number of ships that could be maintained in period \( j \). The notation \( B_i \) relates to the number of periods needed to maintain ship \( i \) at the first visit. The structure of the transportation tableau utilized for solving the preventive maintenance scheduling problem is shown in Table 1.

The horizontal movement indicates period progression while the vertical movement across the table shows the identity by ship. Take the element \( y_{ij} \) which could be assigned 0 or 1. For all the values of \( y_{ij} \) equal to 1, the sum must be equal to, greater or less than \( h \) along every row, relating to a particular ship. \( x \) is the planned period for which maintenance and operational activities are carried out. \( S \) represents the surplus for each period while \( A_j \) is the capacity constraint.

**Step-by-step approach in solving the problem:** Having stated the problem, the approach in solving it is as detailed below:

**Step 1.** Obtain the entry parameters (i.e. operations period, maintenance periods, arrival periods, maintenance capacity, period-dependent cost, total number of machines to be maintained, total periods of maintenance and the number of machines) designated as: \( O_i', B_i', K_i', A_j \), \( C_{ij}, M, T \) and \( N \) respectively.

**Step 2.** Develop the transportation tableau by: (a) Indicating the values of the objective function cost and positions where \( y_{ij} \) are 1. The function costs are indicated in the boxes while the values of \( y_{ij} \) are stated under the boxes. (b) Based on the values of \( B_i \) and \( A_j \) (which are stated along the vertical and horizontal columns respectively) allocations of all the \( y_{ij} \) are made. (c) The sub-cost for ship is then computed by multiplying the values in the boxes by the \( y_{ij} \) values (i.e. 1). (d) Sum up all these costs to make up the total cost.

**Step 3.** Set up the table that indicates the ship maintenance, operations and idle periods (months): (a) Idleness is calculated from the transportation tableau by observing when the ship starts maintenance and its discontinuities. These discontinuities of periods of maintenance are added up as the idle time for the ship; (b) The maintenance period is read as \( B_i \); (c) The operation period is then obtained from the subtraction of the idle and maintenance periods from the total available periods; (d) The sum, mean and standard deviations of the idle and operation periods are then obtained.

**Step 4.** Set up the cost of the schedule either in the inflationary or non-inflationary period: (a) Cost from the tableau is obtained as the sub-
total of costs indicated in step 2(c). (b) Cost of idleness is then calculated based on the knowledge of the revenue losses of ships per unit period of analysis. (c) Cost of schedule is obtained as the sum of cost from the tableau and the cost of idleness.

**Step 5.** Obtain the table of functional minimization versus actual costs in the inflationary and non-inflationary conditions (a) List all the formulations along both the vertical and horizontal axes.

### 3. Case study

The case study presented here is a shipping organization that is based in Lagos, Nigeria. In this paper, its name is referred to as Dynamics Nigeria Limited (DNL). The challenge for the organization is to be able to control its maintenance workforce who may be relaxed at implementing maintenance task in a timely manner. The control, which involves attacking a cost implication to delays of ships at the dockyard is targeted at a high turnover of maintenance services. DNL has a successful operational record of transporting goods outside Nigeria and importing other goods into Nigeria from distant and near countries.

Crude oil transportation to refineries outside Nigeria is a major activity that the shipping company is engaged in. It also imports petroleum products from these refineries into the country. The ship maintenance activities include sand blasting, welding of damaged ship parts and bodies, pump servicing, paint spraying, generator servicing, valve servicing, fixing stainers, radar repair, engine repair, ship reconstruction, propeller fabrication, rudder fabrication, electrical system maintenance, brake system maintenance, etc.

The Chief Executive of this shipping organization controls both the administrative and operational activities of the firm. However, he is responsible to the Board of Directors that is constituted from both within and outside the organization. The organization is structured into 5 sections: Accounts and Budgets, Logistics, Materials, Operations and Personnel. The Accounts and Budgets department prepares estimates of current and recurrent expenditures for the company’s activities. The Logistics department provides procurement, installation, and maintenance of all equipment and facilities and their spares. The Materials Department is engaged in the utilization of the materials. The operations department is responsible for the daily operations and training. The Personnel department recruits for the organization. In order to obtain reliable data used in this paper, two main approaches were adopted. The first concerns historical records collected from the Accounting and Engineering units as well as the dockyard where actual maintenance of ships are carried out. The second approach is the information gathered from interviews with all levels of staff in the organization. Using the second approach, both direct and indirect questions were posed to administrative staff, engineering employees and craftsmen. Information obtained through instructions was validated by ensuring that supporting data are sighted. However, some difficulties were encountered in doing this, primarily, the reluctance of some personnel in revealing vital information for the study.

### 4. Results

Table 2 illustrates the ships-periods final transportation tableau matrix with $y_{ij}$ indicated. This is shown only for the new method. By following the same procedure, results were obtained that show values for the old method. However, the final values are used in this section. Table 3 shows the preliminary cost minimization for the old and new methods. In Table 4, a summary of ship maintenance, operation and idle period are provided in months with some statistical measures of mean and standard deviation obtained for the old and new methods. Table 5 shows ship’s description and preventive maintenance data. This includes the ship identity, the maximum running time, sizes of the ships, maximum passengers allowed on board, type of operation, tonnage and arrival period of ships at the dockyard. Table 6 shows the computation of the cost of schedule using the old and new methods. This includes cost from the tableau, idleness and opportunity costs.

### 5. Discussion

The data analysis for the study principally hinges on the platform of the transportation model presented in the section on modeling. The analysis of data shall be approached here from the perspective of treating the components of the transportation tableau needed to achieve results. This involves four basic steps: (1) Computation of period-dependent cost ($a_t$); (2) Computation of opportunity cost; (3) Description of allocation of maintenance period in the ships-periods final transportation tableau matrix with $Y_{ij}$ indicated; (4) Computation of cost of schedule for the tradvar procedure. These are shown in the next subsections.
Table 1. Structure of the ships-periods final transportation tableau matrix.

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Table 2. Ships-periods final transportation tableau matrix with $y_{ij}$ indicated (New Method).

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Table 2 (continued). Ships-periods final transportation tableau matrix with $y_{ij}$ indicated (New Method).

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<td>$\propto$</td>
<td>0.1</td>
<td>0.2</td>
<td>2 (0)</td>
</tr>
<tr>
<td>$S_i$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$A_j$</td>
<td>3 (1)</td>
<td>3</td>
<td>3</td>
<td>3 (0)</td>
<td>3 (0)</td>
<td>3 (0)</td>
<td>3 (2)</td>
<td>3</td>
<td>3</td>
<td>3 (0)</td>
<td>3 (0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3. Preliminary cost computation.

<table>
<thead>
<tr>
<th>Problem definition: Maintenance cost minimization ($\Delta$)</th>
<th>Sub-total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ship 1 (New) ($1 \times 0.5$) + ($1 \times 0.4$) + ($1 \times 0.2$) + ($1 \times 0.2$)</td>
<td>1.3</td>
</tr>
<tr>
<td>Ship 1 (Old) ($1 \times 0.5$) + ($1 \times 0.2$) + ($1 \times 0.2$) + ($1 \times 0.1$)</td>
<td>1.0</td>
</tr>
<tr>
<td>Ship 2 (New) ($1 \times 0.2$) + ($1 \times 0.3$) + ($1 \times 0.1$) + ($1 \times 0.2$)</td>
<td>0.8</td>
</tr>
<tr>
<td>Ship 2 (Old) ($1 \times 0.3$) + ($1 \times 0.3$) + ($1 \times 0.2$) + ($1 \times 0.1$)</td>
<td>0.9</td>
</tr>
<tr>
<td>Ship 3 (New) ($1 \times 0.1$) + ($1 \times 0.1$) + ($1 \times 0.1$) + ($1 \times 0.1$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Ship 3 (Old) ($1 \times 0.1$) + ($1 \times 0.1$) + ($1 \times 0.1$) + ($1 \times 0.1$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Ship 4 (New) ($1 \times 2.1$) + ($1 \times 0.1$) + ($1 \times 0.2$)</td>
<td>2.4</td>
</tr>
<tr>
<td>Ship 4 (Old) ($1 \times 2.1$) + ($1 \times 1.3$) + ($1 \times 0.2$)</td>
<td>3.6</td>
</tr>
<tr>
<td>Ship 5 (New) ($1 \times 0.3$) + ($1 \times 0.2$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Ship 5 (Old) ($1 \times 0.3$) + ($1 \times 0.3$)</td>
<td>0.6</td>
</tr>
<tr>
<td>Ship 6 (New) ($1 \times 0.3$) + ($1 \times 0.3$)</td>
<td>0.6</td>
</tr>
<tr>
<td>Ship 6 (Old) ($1 \times 0.6$) + ($1 \times 1.0$)</td>
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</tr>
<tr>
<td>Ship 7 (New) ($1 \times 0.5$) + ($1 \times 0.3$)</td>
<td>0.8</td>
</tr>
<tr>
<td>Ship 7 (Old) ($1 \times 0.5$) + ($1 \times 0.3$)</td>
<td>0.8</td>
</tr>
<tr>
<td>Ship 8 (New) ($1 \times 0.2$) + ($1 \times 0.3$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Ship 8 (Old) ($1 \times 0.5$) + ($1 \times 0.4$)</td>
<td>0.9</td>
</tr>
</tbody>
</table>

This gives a total of $\Delta 7.4$ million (New) and $\Delta 9.2$ million (Old).
Table 4. Ship maintenance, operation, and idle periods (months).

<table>
<thead>
<tr>
<th>Ship (1)</th>
<th>Idleness (New) (2)</th>
<th>Idleness (Old) (3)</th>
<th>Maintenance (4)</th>
<th>Operation Periods (New) (5) = 24 – [(2) + (4)]</th>
<th>Operation Periods (Old) (6) = 24 – [(3) + (4)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>16</td>
<td>4</td>
<td>8</td>
<td>4</td>
</tr>
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<td>13</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>20</td>
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<td>5</td>
<td>2</td>
<td>4</td>
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<td>20</td>
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<td>6</td>
<td>8</td>
<td>8</td>
<td>2</td>
<td>14</td>
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<td>7</td>
<td>9</td>
<td>11</td>
<td>2</td>
<td>13</td>
<td>11</td>
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<td>8</td>
<td>13</td>
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</tr>
<tr>
<td>Total</td>
<td>74</td>
<td>61</td>
<td></td>
<td>94</td>
<td>107</td>
</tr>
<tr>
<td>Mean</td>
<td>9.25</td>
<td>7.63</td>
<td></td>
<td>11.75</td>
<td>13.78</td>
</tr>
<tr>
<td>SD</td>
<td>5.34</td>
<td>4.31</td>
<td></td>
<td>6.04</td>
<td>4.63</td>
</tr>
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</table>

Note: SD = Standard Deviation.

Table 5. Ship's description and preventive maintenance data.

<table>
<thead>
<tr>
<th>Ship</th>
<th>Max. running Time</th>
<th>Size</th>
<th>Max. passengers allowed on board</th>
<th>Type of operation</th>
<th>Tonnage</th>
<th>Arrival (k_i) period</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80</td>
<td>Large</td>
<td>70</td>
<td>Oil carrier</td>
<td>1500</td>
<td>01</td>
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<tr>
<td>2</td>
<td>75</td>
<td>Large</td>
<td>70</td>
<td>Cargo transport</td>
<td>1800</td>
<td>06</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>Medium</td>
<td>25</td>
<td>Cargo transport</td>
<td>800</td>
<td>06</td>
</tr>
<tr>
<td>4</td>
<td>77</td>
<td>Large</td>
<td>70</td>
<td>Cargo transport</td>
<td>1700</td>
<td>20</td>
</tr>
<tr>
<td>5</td>
<td>57</td>
<td>Large</td>
<td>70</td>
<td>Oil carrier</td>
<td>1900</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>Large</td>
<td>70</td>
<td>Oil carrier</td>
<td>1650</td>
<td>10</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>Large</td>
<td>65</td>
<td>Cargo transport</td>
<td>2000</td>
<td>09</td>
</tr>
<tr>
<td>8</td>
<td>70</td>
<td>Large</td>
<td>70</td>
<td>Cargo transport</td>
<td>1600</td>
<td>04</td>
</tr>
</tbody>
</table>

Table 6. Computation of cost of schedule for the problem (₦ million).

<table>
<thead>
<tr>
<th>Ship</th>
<th>Cost from tradvar procedure (tableau)</th>
<th>Cost of Idleness</th>
<th>Opportunity cost of ship in maintenance</th>
<th>Cost of schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New (1)</td>
<td>Old (2)</td>
<td>New (3)</td>
<td>Old (4)</td>
</tr>
<tr>
<td>1</td>
<td>1.3</td>
<td>1.0</td>
<td>9</td>
<td>12.0</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>0.9</td>
<td>11.25</td>
<td>5.25</td>
</tr>
<tr>
<td>3</td>
<td>0.5</td>
<td>0.5</td>
<td>10.50</td>
<td>4.5</td>
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<tr>
<td>4</td>
<td>2.4</td>
<td>3.6</td>
<td>0.75</td>
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<td>0.5</td>
<td>0.6</td>
<td>1.50</td>
<td>3.0</td>
</tr>
<tr>
<td>6</td>
<td>0.6</td>
<td>0.9</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
<td>0.8</td>
<td>6.75</td>
<td>8.25</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.9</td>
<td>9.75</td>
<td>5.25</td>
</tr>
<tr>
<td>Total</td>
<td>7.4</td>
<td>9.2</td>
<td>55.5</td>
<td>45.75</td>
</tr>
</tbody>
</table>
Table 7. Computation of period-dependent cost \((a_i)\) Year 1.

<table>
<thead>
<tr>
<th>Description</th>
<th>Jan ((\Delta))</th>
<th>Feb ((\Delta))</th>
<th>Mar ((\Delta))</th>
<th>Apr ((\Delta))</th>
<th>May ((\Delta))</th>
<th>Jun ((\Delta))</th>
<th>Jul ((\Delta))</th>
<th>Aug ((\Delta))</th>
<th>Sep ((\Delta))</th>
<th>Oct ((\Delta))</th>
<th>Nov ((\Delta))</th>
<th>Dec ((\Delta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Servicing</td>
<td>192000</td>
<td>168000</td>
<td>97500</td>
<td>280000</td>
<td>405000</td>
<td>180000</td>
<td>480000</td>
<td>390000</td>
<td>50000</td>
<td>130000</td>
<td>60000</td>
<td>97500</td>
</tr>
<tr>
<td>Valve servicing</td>
<td>600</td>
<td>700</td>
<td>250</td>
<td>260</td>
<td>855</td>
<td>798</td>
<td>243</td>
<td>150</td>
<td>1440</td>
<td>525</td>
<td>800</td>
<td>170</td>
</tr>
<tr>
<td>Fixing strainers</td>
<td>200</td>
<td>135</td>
<td>120</td>
<td>225</td>
<td>200</td>
<td>300</td>
<td>30</td>
<td>125</td>
<td>140</td>
<td>100</td>
<td>60</td>
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<td>180000</td>
<td>280000</td>
<td>112000</td>
<td>638000</td>
<td>105000</td>
<td>72000</td>
<td>144000</td>
<td>117000</td>
<td>300000</td>
<td>264000</td>
<td>324000</td>
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<td>136000</td>
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<td>220000</td>
<td>180000</td>
<td>600000</td>
<td>90000</td>
<td>330000</td>
<td>180000</td>
<td>140000</td>
<td>135000</td>
<td>780000</td>
</tr>
<tr>
<td>Weld area</td>
<td>475000</td>
<td>89000</td>
<td>364000</td>
<td>258000</td>
<td>292000</td>
<td>550000</td>
<td>306000</td>
<td>172500</td>
<td>240000</td>
<td>315000</td>
<td>360000</td>
<td>332500</td>
</tr>
<tr>
<td>Pump servicing</td>
<td>5200</td>
<td>108000</td>
<td>335400</td>
<td>80000</td>
<td>350000</td>
<td>270750</td>
<td>240000</td>
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<td>113400</td>
<td>630000</td>
<td>450000</td>
<td>792000</td>
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<td>Engine repair and</td>
<td>1800</td>
<td>9100</td>
<td>4500</td>
<td>9200</td>
<td>6300</td>
<td>8750</td>
<td>11900</td>
<td>20000</td>
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<td>7380</td>
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<td>300</td>
<td>325</td>
<td>336</td>
<td>150</td>
<td>154</td>
<td>660</td>
<td>266</td>
<td>448</td>
<td>855</td>
<td>459</td>
</tr>
<tr>
<td>Propeller</td>
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<td>7800</td>
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<td>5558</td>
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<td>1390302</td>
<td>1005970</td>
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<td>1776343</td>
<td>1667605</td>
<td>1806476</td>
<td>1597329</td>
<td>1322329</td>
<td>2388824</td>
<td>2388824</td>
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</tbody>
</table>

Table 8. Computation of period-dependent cost \((a_i)\) Year 2.

<table>
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<th>Description</th>
<th>Jan ((\Delta))</th>
<th>Feb ((\Delta))</th>
<th>Mar ((\Delta))</th>
<th>Apr ((\Delta))</th>
<th>May ((\Delta))</th>
<th>Jun ((\Delta))</th>
<th>Jul ((\Delta))</th>
<th>Aug ((\Delta))</th>
<th>Sep ((\Delta))</th>
<th>Oct ((\Delta))</th>
<th>Nov ((\Delta))</th>
<th>Dec ((\Delta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Servicing</td>
<td>216750</td>
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<td>225000</td>
<td>189000</td>
<td>120000</td>
<td>168000</td>
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<td>182000</td>
<td>108500</td>
<td>105000</td>
<td>75000</td>
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<td>Valve servicing</td>
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<td>260</td>
<td>1550</td>
<td>72</td>
<td>882</td>
<td>324</td>
<td>675</td>
<td>250</td>
<td>190</td>
<td>150</td>
<td>700</td>
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<tr>
<td>Fixing strainers</td>
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<td>125</td>
<td>175</td>
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<td>200</td>
<td>375</td>
<td>100</td>
<td>45</td>
<td>60</td>
<td>120</td>
<td>450</td>
<td>112</td>
</tr>
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<td>190000</td>
<td>297000</td>
<td>297000</td>
<td>672000</td>
<td>319000</td>
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<td>240000</td>
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<td>330000</td>
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<td>627000</td>
<td>133000</td>
<td>225000</td>
<td>780000</td>
<td>150000</td>
<td>840000</td>
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<td>240000</td>
<td>550000</td>
<td>172500</td>
<td>137500</td>
<td>255000</td>
<td>220000</td>
<td>390000</td>
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<td>99000</td>
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<td>306000</td>
<td>240000</td>
<td>540000</td>
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<td>Engine repair and</td>
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<td>19000</td>
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<td>2400</td>
<td>1800</td>
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<td>9200</td>
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<td>6300</td>
<td>4500</td>
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<td>513</td>
<td>570</td>
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<td>510</td>
<td>540</td>
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<td>7000</td>
<td>11050</td>
<td>7200</td>
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<td>1750</td>
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<td>14952</td>
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<td>1945868</td>
<td>1294620</td>
<td>1987557</td>
<td>1938704</td>
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<td>1662138</td>
<td>923820</td>
<td>1688084</td>
</tr>
</tbody>
</table>

Note: \(\Delta 150 = \$1\)
Table 9. Cost computation for preventive maintenance scheduling cost of ships.

<table>
<thead>
<tr>
<th>Ship 1</th>
<th>Preliminary cost computation</th>
<th>Cost from tradvar procedure (tableau)</th>
<th>Idleness</th>
<th>Maintenance</th>
<th>Operation period</th>
<th>Cost of idleness (7 = (4) \times 0.75)</th>
<th>Opportunity cost of ship in maintenance (8 = (5) \times 0.75)</th>
<th>Cost of schedule (9 = (3) + (7) + (8))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original value</td>
<td>((1 \times 0.5) + (1 \times 0.4) + (1 \times 0.2) = 1.3)</td>
<td>12 4 8 9.0 3.0 13.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>33.3% increase in (A_1)</td>
<td>((1 \times 0.5) + (1 \times 0.4) + (1 \times 0.2) = 1.3)</td>
<td>12 4 8 9.0 3.0 13.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>66.7% increase in (A_1)</td>
<td>((1 \times 0.5) + (1 \times 0.4) + (1 \times 0.2) = 1.3)</td>
<td>12 4 8 9.0 3.0 13.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100% increase in (A_1)</td>
<td>((1 \times 0.5) + (1 \times 0.4) + (1 \times 0.2) = 1.3)</td>
<td>12 4 8 9.0 3.0 13.3</td>
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<tr>
<td></td>
<td>133.3% increase in (A_1)</td>
<td>((1 \times 0.5) + (1 \times 0.4) + (1 \times 0.2) = 1.3)</td>
<td>12 4 8 9.0 3.0 13.3</td>
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<tr>
<td></td>
<td>166.7% increase in (A_1)</td>
<td>((1 \times 0.5) + (1 \times 0.4) + (1 \times 0.2) = 1.3)</td>
<td>12 4 8 9.0 3.0 13.3</td>
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<tr>
<td>Ship 2</td>
<td>Original value</td>
<td>((1 \times 0.2) + (1 \times 0.1) = 0.8)</td>
<td>15 4 5 11.25 3.0 15.05</td>
<td></td>
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<tr>
<td></td>
<td>33.3% increase in (A_2)</td>
<td>((1 \times 0.2) + (1 \times 0.1) = 0.8)</td>
<td>15 4 5 11.25 3.0 15.05</td>
<td></td>
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<tr>
<td></td>
<td>66.7% increase in (A_2)</td>
<td>((1 \times 0.2) + (1 \times 0.1) = 0.8)</td>
<td>15 4 5 11.25 3.0 15.05</td>
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<tr>
<td></td>
<td>100% increase in (A_2)</td>
<td>((1 \times 0.2) + (1 \times 0.1) = 0.8)</td>
<td>15 4 5 11.25 3.0 15.05</td>
<td></td>
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<tr>
<td></td>
<td>133.3% increase in (A_2)</td>
<td>((1 \times 0.2) + (1 \times 0.1) = 0.8)</td>
<td>15 4 5 11.25 3.0 15.05</td>
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<tr>
<td></td>
<td>166.7% increase in (A_2)</td>
<td>((1 \times 0.2) + (1 \times 0.1) = 0.8)</td>
<td>15 4 5 11.25 3.0 15.05</td>
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<tr>
<td>Ship 3</td>
<td>Original value</td>
<td>((1 \times 0.1) + (1 \times 0.1) + (1 \times 0.1) = 0.5)</td>
<td>14 5 5 10.50 3.75 14.75</td>
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<tr>
<td></td>
<td>33.3% increase in (A_3)</td>
<td>((1 \times 0.1) + (1 \times 0.1) + (1 \times 0.1) = 0.5)</td>
<td>14 5 5 10.50 3.75 14.75</td>
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<tr>
<td></td>
<td>66.7% increase in (A_3)</td>
<td>((1 \times 0.1) + (1 \times 0.1) + (1 \times 0.1) = 0.5)</td>
<td>14 5 5 10.50 3.75 14.75</td>
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<tr>
<td></td>
<td>100% increase in (A_3)</td>
<td>((1 \times 0.1) + (1 \times 0.1) + (1 \times 0.1) = 0.5)</td>
<td>14 5 5 10.50 3.75 14.75</td>
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<tr>
<td></td>
<td>133.3% increase in (A_3)</td>
<td>((1 \times 0.1) + (1 \times 0.1) + (1 \times 0.1) = 0.5)</td>
<td>14 5 5 10.50 3.75 14.75</td>
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<tr>
<td></td>
<td>166.7% increase in (A_3)</td>
<td>((1 \times 0.1) + (1 \times 0.1) + (1 \times 0.1) = 0.5)</td>
<td>14 5 5 10.50 3.75 14.75</td>
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<tr>
<td>Ship 4</td>
<td>Original value</td>
<td>((1 \times 2.1) + (1 \times 0.1) + (1 \times 0.2) = 2.4)</td>
<td>2 3 19 1.50 2.25 6.15</td>
<td></td>
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<tr>
<td></td>
<td>33.3% increase in (A_4)</td>
<td>((1 \times 2.1) + (1 \times 0.1) + (1 \times 0.2) = 2.4)</td>
<td>2 3 19 1.50 2.25 6.15</td>
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</tr>
<tr>
<td></td>
<td>66.7% increase in (A_4)</td>
<td>((1 \times 2.1) + (1 \times 0.1) + (1 \times 0.2) = 2.4)</td>
<td>2 3 19 1.50 2.25 6.15</td>
<td></td>
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<tr>
<td></td>
<td>100% increase in (A_4)</td>
<td>((1 \times 2.1) + (1 \times 0.1) + (1 \times 0.2) = 2.4)</td>
<td>2 3 19 1.50 2.25 6.15</td>
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<tr>
<td></td>
<td>133.3% increase in (A_4)</td>
<td>((1 \times 2.1) + (1 \times 0.1) + (1 \times 0.2) = 2.4)</td>
<td>2 3 19 1.50 2.25 6.15</td>
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<tr>
<td></td>
<td>166.7% increase in (A_4)</td>
<td>((1 \times 2.1) + (1 \times 0.1) + (1 \times 0.2) = 2.4)</td>
<td>2 3 19 1.50 2.25 6.15</td>
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</tr>
<tr>
<td>Ship 5</td>
<td>Original value</td>
<td>((1 \times 0.3) + (1 \times 0.2) = 0.5)</td>
<td>2 2 20 1.50 1.50 3.50</td>
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<tr>
<td></td>
<td>33.3% increase in (A_5)</td>
<td>((1 \times 0.3) + (1 \times 0.1) = 0.4)</td>
<td>8 2 14 6.0 1.50 7.90</td>
<td></td>
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<tr>
<td></td>
<td>66.7% increase in (A_5)</td>
<td>((1 \times 0.1) + (1 \times 0.2) = 0.3)</td>
<td>9 2 13 6.75 1.50 8.55</td>
<td></td>
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<tr>
<td></td>
<td>100% increase in (A_5)</td>
<td>((1 \times 0.1) + (1 \times 0.2) = 0.3)</td>
<td>9 2 13 6.75 1.50 8.55</td>
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<tr>
<td></td>
<td>133.3% increase in (A_5)</td>
<td>((1 \times 0.1) + (1 \times 0.2) = 0.3)</td>
<td>9 2 13 6.75 1.50 8.55</td>
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</tr>
</tbody>
</table>

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The table above represents the cost computation for preventive maintenance scheduling cost of ships, using the Preliminary cost computation method and the Cost from tradvar procedure (tableau) method. Each row represents a different ship with details on the original value and the cost of idleness, opportunity cost of ship in maintenance, and the cost of schedule. The percentages denote the increase in each category from the original value.
Table 9 (continued). Cost computation for preventive maintenance scheduling cost of ships.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Preliminary cost computation</th>
<th>Idleness</th>
<th>Maintenance</th>
<th>Operation period</th>
<th>Cost of idleness</th>
<th>Opportunity cost of ship in maintenance</th>
<th>Cost of schedule</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cost from tradvar procedure (tableau)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Ship 6</td>
<td>Original value</td>
<td>(1 x 0.3) + (1 x 0.3) = 0.6</td>
<td>8 2 14</td>
<td>6.00 1.50</td>
<td>8.10</td>
<td></td>
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<tr>
<td></td>
<td>33.3% increase in $A_i$</td>
<td>(1 x 0.3) + (1 x 0.2) = 0.5</td>
<td>6 2 16</td>
<td>4.50 1.50</td>
<td>6.00</td>
<td></td>
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<tr>
<td></td>
<td>66.7% increase in $A_i$</td>
<td>(1 x 0.3) + (1 x 0.1) = 0.4</td>
<td>12 2 10</td>
<td>9.0 1.50</td>
<td>10.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>100% increase in $A_i$</td>
<td>(1 x 0.3) + (1 x 0.1) = 0.4</td>
<td>12 2 10</td>
<td>9.0 1.50</td>
<td>10.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>133.3% increase in $A_i$</td>
<td>(1 x 0.1) + (1 x 0.2) = 0.3</td>
<td>13 2 9</td>
<td>9.75 1.50</td>
<td>11.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>166.7% increase in $A_i$</td>
<td>(1 x 0.1) + (1 x 0.2) = 0.3</td>
<td>13 2 9</td>
<td>9.75 1.50</td>
<td>11.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ship 7</td>
<td>Original value</td>
<td>(1 x 0.5) + (1 x 0.3) = 0.8</td>
<td>9 2 13</td>
<td>6.75 1.50</td>
<td>8.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>33.3% increase in $A_i$</td>
<td>(1 x 0.3) + (1 x 0.2) = 0.5</td>
<td>7 2 15</td>
<td>5.25 1.50</td>
<td>6.75</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>66.7% increase in $A_i$</td>
<td>(1 x 0.3) + (1 x 0.2) = 0.5</td>
<td>7 2 15</td>
<td>5.25 1.50</td>
<td>6.75</td>
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<tr>
<td></td>
<td>100% increase in $A_i$</td>
<td>(1 x 0.3) + (1 x 0.2) = 0.5</td>
<td>7 2 15</td>
<td>5.25 1.50</td>
<td>6.75</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>133.3% increase in $A_i$</td>
<td>(1 x 0.1) + (1 x 0.3) = 0.4</td>
<td>14 2 8</td>
<td>10.50 1.50</td>
<td>12.00</td>
<td></td>
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<tr>
<td></td>
<td>166.7% increase in $A_i$</td>
<td>(1 x 0.1) + (1 x 0.3) = 0.4</td>
<td>14 2 8</td>
<td>10.50 1.50</td>
<td>12.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ship 8</td>
<td>Original value</td>
<td>(1 x 0.2) + (1 x 0.3) = 0.5</td>
<td>13 2 9</td>
<td>9.75 1.50</td>
<td>11.25</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>33.3% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.3) = 0.5</td>
<td>13 2 9</td>
<td>9.75 1.50</td>
<td>11.25</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>66.7% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.2) = 0.4</td>
<td>12 2 10</td>
<td>9.0 1.50</td>
<td>10.50</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>100% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.2) = 0.4</td>
<td>12 2 10</td>
<td>9.0 1.50</td>
<td>10.50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>133.3% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.3) = 0.5</td>
<td>11 2 11</td>
<td>8.25 1.50</td>
<td>10.25</td>
<td></td>
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<tr>
<td></td>
<td>166.7% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.3) = 0.5</td>
<td>11 2 11</td>
<td>8.25 1.50</td>
<td>10.25</td>
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<tr>
<td></td>
<td>Original value</td>
<td>(1 x 0.2) + (1 x 0.3) = 0.5</td>
<td>13 2 9</td>
<td>9.75 1.50</td>
<td>11.25</td>
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<tr>
<td></td>
<td>33.3% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.3) = 0.5</td>
<td>13 2 9</td>
<td>9.75 1.50</td>
<td>11.25</td>
<td></td>
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<tr>
<td></td>
<td>66.7% increase in $A_i$</td>
<td>(1 x 0.2) + (1 x 0.2) = 0.4</td>
<td>12 2 10</td>
<td>9.0 1.50</td>
<td>10.50</td>
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</table>

An opportunity cost maintenance scheduling framework . . . 73
5.1. Computation of period-dependent cost \( (a_i) \)

For the shipping industry considered, the peculiarity of the maintenance cost \( (a_i) \) makes it to be composed of several cost components, which may only be found in the shipping system. The cost involved here includes those due to generator servicing, valve servicing, fixing strainers, sand blasting, paint spray, weld area, pump servicing, engine repair and servicing, shaft, propeller, rudder and docking costs. The aggregate of these costs makes up the period-dependent cost \( (a_i) \). For example, in January of Year 1 of analysis, the generator servicing cost was \( \text{₦} 192,000 \), sand blasting cost was \( \text{₦} 684,000 \), paint spray cost was \( \text{₦} 660,000 \) and other associated costs which make up \( \text{₦} 2,089,390 \). For February of year 1, the total cost was \( \text{₦} 721,859 \). All other costs are as shown in Table 8.

5.2. Computation of opportunity cost \( (d_i) \)

In order to compute the opportunity cost used for analysis in this work, two approaches were adopted and a reasonable minimum value of the choices was adopted in the computation of the total preventive maintenance cost utilized in the study. The first approach considered price charged for commercial activities for the various tonnage of ships. The second approach is the use of depreciation value of the particular ship of interest.

These two approaches aim at obtaining the market values of the services rendered by the ship per period. From investigation and proper analysis, it was observed that \( \text{₦} 2.5 \) million is charged for two weeks for an oil carrier vessel. This is a possible opportunity cost.

An alternative is to consider the depreciation cost, which is the minimum cost incurred as the opportunity cost. Depreciation cost of ships varies according to tonnage. However, to avoid problem complexity, we would consider an average ship of 1650 tons and use its depreciation cost to generalize for the ships. This ship cost \( \text{₦} 15 \) million with an expected life span of twenty years.

Thus, the average annual depreciation cost is \( \text{₦} 15 \) million / 20 years = \( \text{₦} 750,000 \). Now, considering the values of these two costs, the minimum is adopted (i.e. \( \text{₦} 750,000 \)). This is used as the value of the opportunity cost in the computation of the total preventive maintenance cost.

5.3. Description of allocation of maintenance period in the ships-periods final transportation tableau matrix with \( x_{ij} \) indicated

The model utilized in the allocation of maintenance period to a ship is a variant of the techniques of Vogel’s Approximation method, North-east corner rule or others in the sense that allocations are made in contiguous period (one after the other). Thus, consider Table 2 where the entries in the North-east corner of each cell represent the maintenance cost incurred for each ship in the period concerned. The procedure, christened ‘tradvar’ in this work is as follows:

i. The minimum and the maximum values in each row and column is sorted for. This is computed and placed below the maintenance capacity \( (A_j) \) row or after the column representing duration per maintenance visit \( (B_i) \).

ii. The values obtained from (i) forms the first iteration. From Table 2, the first iteration gives set values along the columns as: \( (2.1, 0.7, 1.4, 0, 2.0, 0.7, 0.5, 0.4, 1.3, 0.3, 0.2, 0.8, 1.9, 0.2, 0.1, 0.3, 0.2, 2.1, 1.3, 1.7, 0, 0.2) \).

Along the row, the values obtained from the first iteration are as follows: \( (2.0, 1.1, 0.4, 2.0, 1.8, 1.2, 0.7, 1.9) \).

iii. From this set of values along the row and columns of the first iteration, the minimum value is obtained. This minimum value is for the fourth period along the row of the first iteration. The value is zero. It thus means that allocations should start for the first ship on arrival for all the maintenance period that ship 1 would spend at the dockyard. Thus, the maintenance periods are allocated one after the other. It then means that allocations of maintenance period to ship 1 are done in the periods 4, 12, 13 and 16 respectively. The gap in between these allocations is due to the fact that the intermittent periods have invisible allocation values. This corresponds to the ‘Big M’ concept in the simplex method of solving problems under linear programming techniques.

iv. Having done allocations for ship 1, it means that the duration per maintenance visit \( (B_i) \) is exhausted for ship 1, hence no allocation
could be made for ship 1 anymore. This leads us to the second iteration. The procedure followed for subsequent iterations is similar to that carried out for iteration 1.

v. Having following all the necessary iterations, final allocations are made for all the entries as shown in Table 2. In all, 14 iterations were made.

5.4. Computation of cost of schedule for the tradvar procedure

In computing the cost schedule (Table 2) for the tradvar procedure three stages of computation are carried out. The first relates to the preliminary cost computation in which the cost for each ship is determined according to the ship maintenance periods. Here, there is a unitary multiplication with each of the cost units for the period-dependent cost (Table 1). The second stage involves the computation of ship maintenance, operations and idle periods. This information (Table 2) reveals the idle periods for every ship and the total for all ships. The maintenance period for every ship is also indicated. In addition, computation of the operation period for every ship is also shown. Furthermore, the total operation periods for every ship, mean idle times and operation periods, as well as standard deviation for idleness and operation periods are all shown in this table (Table 2). The third phase shows details of cost of schedule for all ships. The components are mainly cost from tradvar procedure (tableau), and cost of idleness. Thus, all computations relating to cost of schedule are shown in Tables 1 to 6 below. It is worth noting that the opportunity cost is being reflected by the level of idleness as well as the loss in revenue on ships due to maintenance. A sensitivity test of the model obtained is shown in Table 9.

6. Conclusion

In this paper, the limitation of the current approach of deriving maintenance schedules without consideration for the operational revenue losses due to idleness or maintenance has been stated. Consequently, the paper intends to develop a mathematical model that incorporates opportunity cost, which represents the amount of revenue loss as an alternative foregone. The tradvar procedure is then utilized to compute the total maintenance scheduling cost. This approach involves determination of costs in which maintenance activities are done in contiguous periods (one after the other). The results obtained from this new approach are then compared with that of the traditional transportation approach in which allocations for maintenance periods need not be contiguous. In addition, several scenarios for changes in the capacity of the dockyard were considered. For example, when the dockyard capacity increases or shrinks by 33.3%, the authors were curious about the results of the analysis. Again, sensitivity analysis was carried out on the maintenance periods where it is desired to know the effects of its increases and decreases on the total maintenance scheduling cost. The practical implication of this is that the maintenance team agrees to spend overtime on duty. It may also be that additional manpower has been absorbed by the shipping organization. Scenarios for the increases of one unit, two units and three units etc for $A_j$ were investigated.

Out of all the results obtained, it was noted that in order to reduce costs and delays, decisions for preventive maintenance scheduling of ships should be based on opportunity costs. This could be achieved when opportunity cost is viewed as a penalty for a team. Since every delay of ships for maintenance at the dockyard involves loss of time and revenue, the accumulated revenue losses for ships could count against a team in performance assessment. Granted that all the team members have responsibility for the losses, they would easily reprimand the ineffective staff that exhibits unsatisfactory attendance behaviour. In other words, opportunity cost concept could be used to check absenteeism in staff. The framework provided may be useful as a foundation research that stimulates quantitative research. It may also assist in improving the overall quality of emerging research. The contribution of the opportunity cost-based maintenance scheduling framework discussed in this work relates to the critical issues of efficiency, effectiveness and organizational competitiveness. In addition, scheduling is made more effective since optimal utilization of human resources is made. Also, optimization of personnel cost, effective implementation of training and retraining activities and maintenance of good employee succession programs are guaranteed. For future studies, it may be interesting to understand the possible effects of incorporating inflation costs into the original framework in order to decide on whether it is useful or not for monitoring costs of preventive maintenance scheduling. It is hoped that the results obtained and the model prescribed in general would be useful to ship management and planners, as well as researchers, and the entire maintenance scheduling community.
Acknowledgement

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References


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