

A multi-objective inventory model for deteriorating items with backorder and stock dependent demand

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Abstract

Classical deterministic inventory models consider the demand rate to be either constant or time-dependent but independent from the stock status. However, for a certain type of inventory, the demand rate may be influenced by the stock level. Also in many real-life problems, some products such as fruits, vegetables, pharmaceuticals and volatile liquids continuously deteriorate to evaporation, obsolescence, spoilage, etc. In this paper, a multi-deteriorating inventory model with shortage in fuzzy form is formulated and solved where the demand's pattern has a linear trend. In this paper, we present a multi-objective inventory model of deteriorating items in fuzzy environment with the consideration of shortage in the problem formulation. The demand here is assumed with a linear trend and the shortage is allowed for all items. The objectives of maximizing net profit of the inventory system and minimizing the total annual cost of deteriorated items are considered subject to the total cost and the storage area. The vagueness in the objectives is expressed by fuzzy linear membership functions and the resulted fuzzy models are transferred into a non-linear programming and solved using Fuzzy Non-Linear Programming (FNLP) method. The implementation of the model is presented with some numerical examples and finally the results of two fuzzy models are compared.

Keywords: Multi-objective programming; Fuzzy inventory; Fuzzy non-linear programming; Deteriorating items

1. Introduction

The concept of Multi-Objective Programming (MOP) has become a popular research topic among many researches during the past few years. Inventory models are developed in fuzzy environments [5,6] expressing the goals and parameters by fuzzy functions and / or fuzzy numbers. Therefore, they may be reduced to fuzzy decision making models solved by different fuzzy programming methods. Yao and Su [14] solve fuzzy inventory model with back order for

fuzzy total demand based on interval-valued fuzzy set. Sakawa and Yano [11] for instance formulate a fuzzy multi-objective problem with fuzzy parameters. Roy and Maiti [9] have examined the fuzzy Economic Order Quality (EOQ) model with demand dependent unit price and imprecise storage by both fuzzy geometric and non-linear programming methods. Mondal and Maiti [7] solved a class of inventory problems in fuzzy environment using fuzzy non-linear programming method based on classical optimization techniques.

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In many real-life situations, some products (e.g., fruits, vegetables, pharmaceuticals, volatile liquids, and others) deteriorate continuously to evaporation, obsolescence, spoilage, etc. Ghare and Schrader [4] studied an inventory model with consideration of the effect of deteriorating items with storage. Covert and Phillip [1], Tadikamalla [13] and Shah [12] extended Ghare and Schrader's model by relaxing the assumption of constant deterioration and introducing other models of deterioration.

Later, Dave and Patel [3] developed an inventory model with deterministic but linearly changing demand rate, constant deterioration rate and finite planning horizon. Sachan [10] extended Dave and Patel's model to allow shortages. Datta and Pall [2] presented an EOQ model considering variables deterioration and power demand pattern. Roy and Maiti [8] developed a multi-objective inventory model considering fuzziness in objectives and constraints along with deteriorating effect on items. In this paper, the researchers have considered the demand pattern as $D_i(q_i) = \alpha_i + \beta_i q_i(t)$ and $a_i q_i(t)$ as the stock dependent deterioration rate. This paper is organized as follows: Some necessary definitions are presented first. The mathematical formulation of the resulted crisp model is presented in the context of a multi-objective deteriorating inventory model with backorder. Next, we use fuzzy concept in our proposed mathematical formulation and using the fuzzy programming concept, the resulted model changes into an ordinary non-linear programming. The implementation of the proposed method in this paper is demonstrated using a numerical example. Finally conclusion remarks are given at the end to summarize the contributions of this paper.

2. Assumptions and notations

A multi-objective inventory model of deteriorating items with infinite rate of replenishment, storage, stock-dependent demand and limited budget is considered with the following assumptions and notations. As we observe in Figure 1, at the beginning of the time cycle, the inventory level is $Q - b$. The on hand stock decreases by demand and deterioration, till inventory reaches zero level. Since in T_2 , there is no stock available, the impact of deterioration and the part of demand which is stock dependent disappears, so the inventory decreases by $-\alpha$.

2.1. Assumptions

- The inventory system involves multi-items and lead time is assumed to be negligible,
- The deterioration occurs only when the items are effectively in stock,
- The demand rate $D_i(q_i)$ is dependent on stock such that $D_i(q_i) = \alpha_i + \beta_i q_i(t)$,
- The deteriorated units are neither repaired nor replaced during the time period T.

2.2. Notations

- Q_i : Order quantity for i th item.
 P_i : Purchasing price of each product of i th item.
 S_i : Selling price for each product of i th item.
 h_i : Holding cost per unit quantity per unit time of i th item.
 π_i : Unit dependent shortage cost of i th item.
 $\hat{\pi}_i$: Time dependent shortage cost of i th item.
 A_i : Set-up cost for i th item.
 a_i : Constant rate of deterioration for i th item, $0 < a_i < 1$.
 T_i : Time period for each cycle of i th item.
 $D_i(q_i)$: Quantity of demand at time $t = \alpha_i + \beta_i q_i(t)$ where β_i and α_i are constant, $0 < \beta_i < 1, \alpha_i > 0$.
 $q_i(t)$: Inventory level at time t of the i th item.
 $TC(Q_i, b_i)$: Total cost of the i th item.
 $NP(Q_i, b_i)$: Total net profit of the i th item.
 $DC(Q_i, b_i)$: Total deteriorating cost of the i th item.
 $E(Q_i, b_i)$: Total amount of deteriorated units for the i th item.
 R : Maximum available storage area.
 C : Maximum available budget.
 r_i : Required area of i th item.

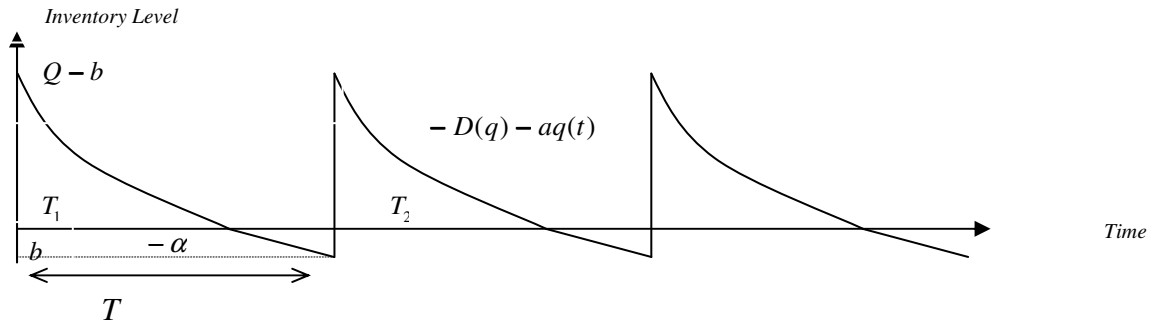


Figure 1. Schematic inventory system.

3. Mathematical formulation

3.1. Crisp model

Suppose $q_i(t)$ is the inventory level at time t of the i th item, therefore,

$$\frac{dq_i}{dt} = \begin{cases} -D_i(q_i) - a_i q_i(t), & 0 \leq t < T_1 \\ -\alpha, & T_1 \leq t < T_1 + T_2 \end{cases} \quad (1)$$

The time length when $0 \leq t < T_1$ is:

$$T_{1i} = \int_0^{T_{1i}} \frac{dq_i}{-D_i(q_i) - a_i q_i(t)} = \int_0^{Q_i - b_i} \frac{dq_i}{\alpha_i + (a_i + \beta_i) q_i(t)} \quad (2)$$

$$= \frac{1}{(a_i + \beta_i)} \ln \left[1 + \left(\frac{a_i + \beta_i}{\alpha_i} \right) (Q_i - b_i) \right].$$

Also the time length when $T_1 \leq t < T_1 + T_2$ is:

$$T_{2i} = \int_0^b \frac{dq_i}{\alpha_i} = \frac{b_i}{\alpha_i}. \quad (3)$$

So the cycle time in each time period is obtained as follows:

$$T_i = T_{1i} + T_{2i}. \quad (4)$$

The holding cost in each cycle for the i th item is:

$$h_i I(Q_i, b_i), \quad (5)$$

where

$$I(Q_i, b_i) = \int_0^{T_{1i}} q_i(t) dt = \int_0^{Q_i - b_i} \frac{q_i(t) dq_i}{\alpha_i + (a_i + \beta_i) q_i(t)} \quad (6)$$

$$= \frac{(Q_i - b_i)}{(a_i + \beta_i)} - \frac{\alpha_i}{(a_i + \beta_i)^2} \ln \left[1 + \left(\frac{a_i + \beta_i}{\alpha_i} \right) (Q_i - b_i) \right].$$

The deteriorating cost in each cycle for the i th item is:

$$DC(Q_i, b_i) = p_i E(Q_i, b_i), \quad (7)$$

where

$$E(Q_i, b_i) = \int_0^{T_{1i}} a_i q_i(t) dt = \int_0^{Q_i - b_i} \frac{a_i q_i(t) dq_i}{D_i(q_i) + a_i q_i(t)} \quad (8)$$

$$a_i I(Q_i, b_i).$$

The shortage cost in each cycle for the i th item is as follows:

$$\pi_i b_i + \hat{\pi}_i B(b_i), \quad (9)$$

where

$$B(b_i) = \int_0^b \frac{q_i(t) dq_i}{\alpha_i} = \frac{b_i^2}{2\alpha_i}. \quad (10)$$

Note that in order to have a feasible solution in (2) and (6), the term in 'ln' should be greater than zero,

so that we need an additional assumption for each item. Therefore,

$$Q_i > b_i - \frac{\alpha_i}{a_i + \beta_i} . \tag{11}$$

The total net profit of the inventory system is:

$$NP(Q, b) = \sum_{i=1}^n NP(Q_i, b_i) = \sum_{i=1}^n [(s_i - p_i)Q_i - h_i I(Q_i, b_i) - \pi_i b_i - \hat{\pi}_i B(b_i) - A_i] / T_i . \tag{12}$$

So, the total annual cost of the inventory system is:

$$TC(Q, b) = \sum_{i=1}^n TC(Q_i, b_i) = \sum_{i=1}^n (p_i Q_i + h_i I(Q_i, b_i) + A_i) / T_i . \tag{13}$$

Hence, the problem is:

$$\begin{aligned} \text{Max } NP(Q, b) &= \sum_{i=1}^n [(s_i - p_i)Q_i - \pi_i b_i - \hat{\pi}_i B(b_i) - h_i I(Q_i, b_i) - A_i] / T_i \\ \text{Min } DC(Q, b) &= \sum_{i=1}^n p_i E(Q_i, b_i) / T_i \end{aligned} \tag{14}$$

Subject to:

$$\begin{aligned} \sum_{i=1}^n TC(Q_i, b_i) &\leq C \\ \sum_{i=1}^n r_i Q_i &\leq R \\ Q_i \geq 0, b_i \geq 0, \quad i &= 1, 2, \dots, n. \end{aligned}$$

3.2 Fuzzy model-I

In this section, the researchers consider model (14) where the objective functions and the budget limit are fuzzy in nature, so the fuzzy model-I is as follows:

$$\begin{aligned} \tilde{M}ax \ NP(Q, b) &= \sum_{i=1}^n [(s_i - p_i)Q_i - \pi_i b_i - \hat{\pi}_i B(b_i) - h_i I(Q_i, b_i) - A_i] / T_i, \\ \tilde{M}in \ DC(Q, b) &= \sum_{i=1}^n p_i E(Q_i, b_i) / T_i \end{aligned} \tag{15}$$

Subject to:

$$\begin{aligned} \sum_{i=1}^n TC(Q_i, b_i) &\leq \tilde{C} \\ \sum_{i=1}^n r_i Q_i &\leq R \\ Q_i \geq 0, b_i \geq 0, \quad i &= 1, 2, \dots, n. \end{aligned}$$

Note that \sim in (15) denotes the fuzzy concept. In fuzzy set theory, the fuzzy objectives and constraints are defined by their membership functions, which are either in linear or non-linear form. So the researchers assume $\mu_{NP}(Q, b)$, $\mu_{DC}(Q, b)$ and $\mu(C)$ as the linear membership functions for the two objectives and the constraint, respectively.

$$\mu_{NP}(Q, b) = \begin{cases} 0 & \text{Case I} \\ 1 + \frac{NP(Q, b) - U_{NP}}{P_{NP}} & \text{Case II} \\ 1 & \text{Case III} \end{cases} \tag{16}$$

Case I : $NP(Q, b) < U_{NP} - P_{NP}$

Case II : $U_{NP} - P_{NP} \leq NP(Q, b) \leq U_{NP}$

Case III : $NP(Q, b) > U_{NP}$

$$\mu_{DC}(Q, b) = \begin{cases} 0 & \text{Case I} \\ 1 - \frac{DC(Q, b) - L_{DC}}{P_{DC}} & \text{Case II} \\ 1 & \text{Case III} \end{cases} \tag{17}$$

Case I : $DC(Q, b) > L_{DC} + P_{DC}$

Case II : $L_{DC} \leq DC(Q, b) \leq L_{DC} + P_{DC}$

Case III : $DC(Q, b) < L_{DC}$

$$\mu(C) = \begin{cases} 0 & C > L_C + P_C \\ 1 - \frac{C - L_C}{P_C} & L_C \leq C \leq L_C + P_C \\ 1 & C < L_C \end{cases} \tag{18}$$

where, P_{NP} is the minimum and P_{DC}, P_C are the maximum accepted violation of the aspiration levels U_{NP} and L_{DC}, L_C , respectively. Now, using the fuzzy non-linear programming technique, the solution of fuzzy multi-objective inventory model can be obtained from:

Max α

Subject to:

$$\begin{aligned}
 1 + \frac{NP(Q, b) - U_{NP}}{P_{NP}} &\geq \alpha \\
 1 - \frac{DC(Q, b) - L_{DC}}{P_{DC}} &\geq \alpha \\
 1 - \frac{C - L_C}{P_C} &\geq \alpha
 \end{aligned} \tag{19}$$

$$\sum_{i=1}^n r_i Q_i \leq R$$

$$Q_i \geq 0, b_i \geq 0$$

$$\alpha \in [0,1], i = 1,2,\dots,n.$$

3.3. Fuzzy model-II

Since in real world situations, it is often very hard to define the costs in crisp form, we prefer to assume those parameters of the objective functions, the technological coefficients and right hand side values of the constraint in fuzzy form to make it more compatible.

$$Max \ NP(Q, b) = \sum_{i=1}^n [(\tilde{s}_i - \tilde{p}_i)Q_i - \tilde{\pi}_i b_i - \tilde{\pi}_i]$$

$$B(b_i) - \tilde{h}_i I(Q_i, b_i) - A_i / T_i$$

$$Min \ DC(Q, b) = \sum_{i=1}^n \tilde{p}_i E(Q_i, b_i) / T_i$$

Subject to: (20)

$$\sum_{i=1}^n T\tilde{C}(Q_i, b_i) \leq \tilde{C}$$

$$\sum_{i=1}^n r_i Q_i \leq R$$

$$Q_i \geq 0$$

$$b_i \geq 0 \quad i = 1,2,\dots,n.$$

Here, the researchers have assumed $\mu(Z)$ and $\mu(Y)$ to be the non-increasing and non-decreasing linear membership functions for fuzzy parameters. Respectively, these are presented as:

$$\mu(Z) = \begin{cases} 0 & Z > L_Z + P_Z \\ 1 - \frac{Z - L_Z}{P_Z} & L_Z \leq Z \leq L_Z + P_Z \\ 1 & Z < L_Z \end{cases} \tag{21}$$

$$\mu(Y) = \begin{cases} 0 & Y < U_Y - P_Y \\ 1 + \frac{Y - U_Y}{P_Y} & U_Y - P_Y \leq Y \leq U_Y \\ 1 & Y > U_Y \end{cases} \tag{22}$$

where (21) is defining the membership functions of cost parameters such as purchasing price, holding cost and shortage costs and (22) presents non-decreasing linear membership function of the selling price.

4. Numerical examples

In this section, the researchers present the implementation of the proposed fuzzy models of this paper using some numerical examples.

4.1 .Results for fuzzy model-I

For numerical illustration, the researchers suppose only two items with input data given in Table 1 with consideration $\tilde{NP} = (350,500)$, $\tilde{DC} = (25,33)$, $\tilde{C} = (1900,2200)$ and $R = 500$.

Table 1. The input data for fuzzy model-I.

| i | α | β | a | s | p | h | π | $\hat{\pi}$ | r | A |
|-----|----------|---------|------|-----|-----|-----|-------|-------------|-----|-----|
| 1 | 100 | 0.3 | 0.05 | 12 | 9 | 1 | 0.6 | 0.9 | 0.5 | 100 |
| 2 | 60 | 0.45 | 0.05 | 15 | 10 | 2 | 1 | 0.5 | 1 | 150 |

Table 2. The optimal solution for fuzzy model-I.

| α | Q_1 | Q_2 | b_1 | b_2 | $TC^*(Q, b)$ | $NP^*(Q, b)$ | $DC^*(Q, b)$ |
|----------|--------|--------|-------|--------|--------------|--------------|--------------|
| 0.516 | 201.08 | 252.43 | 80.96 | 107.48 | 1936.44 | 427.55 | 28.86 |

Table 3. The input data for fuzzy model-II.

| I | α | β | a | \tilde{s} | \tilde{p} | \tilde{h} | $\tilde{\pi}$ | $\tilde{\hat{\pi}}$ | A |
|-----|----------|---------|------|-------------|-------------|-------------|---------------|---------------------|-----|
| 1 | 100 | 0.3 | 0.05 | (12,13) | (8,9) | (0.8,1) | (0.4,0.6) | (0.8,0.9) | 100 |
| 2 | 60 | 0.45 | 0.05 | (15,16) | (7,10) | (1,2) | (0.9,1) | (0.4,0.5) | 150 |

Table 4. The optimal solution for fuzzy model-II.

| α | Q_1 | Q_2 | b_1 | b_2 | $TC^*(Q, b)$ | $NP^*(Q, b)$ | $DC^*(Q, b)$ |
|----------|--------|--------|-------|--------|--------------|--------------|--------------|
| 0.792 | 193.91 | 268.64 | 92.25 | 103.60 | 2610.56 | 468.88 | 26.65 |

The authors have solved the fuzzy model-I using the parameters defined in this part. Table 2 summarizes the results of the implementation of the model-I.

4.2. Results for fuzzy model-II

Fuzzy model-II is numerically illustrated with the following values of inventory parameters in Table 3.

The aspiration level for NP^{\sim} and DC^{\sim} is assumed to be same as model-I, but C is assumed to be $\tilde{C} = (2400,2700)$. The optimal results of this model are presented in Table 4.

5. Sensitivity analysis

5.1. Sensitivity analysis due to $\pi + \hat{\pi}$

In order to study how various shortage costs affect the optimal solution of the multi-item inventory fuzzy model-I, a sensitivity analysis for shortage costs is performed. The values of time dependent shortage cost ($\hat{\pi}$) and unit dependent shortage cost (π) are

changed and the proposed method is solved several times. Table 5 summarizes the effect of variation in π_1 and $\hat{\pi}_1$.

As you can observe, an increase in the shortage costs (time dependent shortage cost ($\hat{\pi}$) and unit dependent shortage cost (π)) results in the decrease of total net profit during each period. On the other hand, increasing the shortage costs, obviously the results will increase the of total annual cost ($TC(Q)$) and also total cost for deteriorated items ($DC(Q)$).

In this case, if the shortage costs are highly greater than holding costs, in order to minimize the total annual cost of the inventory system, the model decreases the amount of backorders in each period. To compare the numerical results for the second item, an analysis has been conducted, the results of which are shown in Table 6.

Table 6 shows the same results for the second item. As it was mentioned earlier, increasing the shortage costs yields increasing the total deteriorating and total costs of inventory system. Figure 2, illustrates the decreasing trend of α for both items due to the increase of $\pi + \hat{\pi}$. In the greater values, α remains without any significant changes.

Table 5. Effect of variation in π_1 and $\hat{\pi}_1$.

| π_1 | $\hat{\pi}_1$ | α | $TC^*(Q, b)$ | $NP^*(Q, b)$ | $DC^*(Q, b)$ |
|---------|---------------|----------|--------------|--------------|--------------|
| 0.1 | 0.1 | 0.971 | 1908.52 | 495.74 | 25.22 |
| 0.2 | 0.2 | 0.840 | 1923.03 | 476.23 | 26.27 |
| 0.3 | 0.3 | 0.745 | 1928.52 | 461.77 | 27.04 |
| 0.4 | 0.4 | 0.669 | 1931.43 | 450.42 | 27.64 |
| 0.5 | 0.5 | 0.607 | 1933.22 | 441.18 | 28.14 |
| 0.6 | 0.6 | 0.556 | 1934.54 | 433.47 | 28.55 |
| 0.7 | 0.7 | 0.513 | 1935.69 | 426.95 | 28.89 |
| 0.8 | 0.8 | 0.475 | 1936.84 | 421.39 | 29.19 |
| 1 | 1 | 0.417 | 1939.36 | 412.55 | 29.66 |
| 1.2 | 1.2 | 0.378 | 1942.43 | 406.08 | 30.00 |
| 1.4 | 1.4 | 0.343 | 1946.13 | 401.48 | 30.25 |
| 1.6 | 1.6 | 0.322 | 1950.56 | 398.43 | 30.41 |
| 1.8 | 1.8 | 0.311 | 1955.85 | 396.75 | 30.51 |
| 2 | 2 | 0.308 | 1961.39 | 396.28 | 30.53 |
| 2.2 | 2.2 | 0.308 | 1961.39 | 396.28 | 30.53 |
| 2.5 | 2.5 | 0.308 | 1961.39 | 396.28 | 30.53 |
| 3 | 3 | 0.308 | 1961.39 | 396.28 | 30.53 |
| 3.5 | 3.5 | 0.308 | 1961.39 | 396.28 | 30.53 |
| 5 | 5 | 0.308 | 1961.39 | 396.28 | 30.53 |

Table 6. Effect of variation in π_2 and $\hat{\pi}_2$.

| π_2 | $\hat{\pi}_2$ | α | $TC^*(Q, b)$ | $NP^*(Q, b)$ | $DC^*(Q, b)$ |
|---------|---------------|----------|--------------|--------------|--------------|
| 0.1 | 0.1 | 0.883 | 1833.60 | 482.46 | 25.93 |
| 0.2 | 0.2 | 0.789 | 1860.87 | 468.40 | 26.68 |
| 0.3 | 0.3 | 0.718 | 1882.52 | 457.75 | 27.25 |
| 0.4 | 0.4 | 0.662 | 1899.74 | 449.33 | 27.70 |
| 0.5 | 0.5 | 0.615 | 1914.15 | 442.34 | 28.07 |
| 0.6 | 0.6 | 0.575 | 1926.58 | 436.35 | 28.39 |
| 0.7 | 0.7 | 0.541 | 1937.57 | 431.13 | 28.67 |
| 0.8 | 0.8 | 0.510 | 1947.41 | 426.52 | 28.91 |
| 1 | 1 | 0.458 | 1964.54 | 418.70 | 29.34 |
| 1.2 | 1.2 | 0.415 | 1972.15 | 412.33 | 29.67 |
| 1.4 | 1.4 | 0.380 | 1991.94 | 407.05 | 29.95 |
| 1.6 | 1.6 | 0.351 | 2003.36 | 402.66 | 30.19 |
| 1.8 | 1.8 | 0.326 | 2013.70 | 398.90 | 30.39 |
| 2 | 2 | 0.306 | 2023.20 | 395.95 | 30.54 |
| 2.2 | 2.2 | 0.289 | 2032.04 | 393.44 | 30.68 |
| 2.5 | 2.5 | 0.270 | 2044.29 | 390.55 | 30.83 |
| 3 | 3 | 0.251 | 2062.92 | 387.73 | 30.99 |
| 3.5 | 3.5 | 0.246 | 2078.16 | 386.99 | 31.02 |
| 5 | 5 | 0.246 | 2078.16 | 386.99 | 31.02 |

In Figures 3 and 4, the variations of both objective functions due to increasing the amount of shortage values are depicted for both items.

According to the obtained results when the total amount of $\pi + \hat{\pi}$ reaches to 4 and 7, relatively for the first and the second item, α and the objective functions remains on a constant rate.

5.2. Sensitivity analysis due to P_{NP}

Different values of P_{NP} are considered and the results of fuzzy model-I are presented in Table 7. As you can see, increasing P_{NP} results a decreasing trend of TC , DC and NP .

5.3. Sensitivity analysis due to P_{DC}

In this part, the results of fuzzy model-I due to variations of P_{DC} are summarized in Table 8. The analysis shows no significant change in NP , TC and DC for great values of P_{DC} .

5.4. Sensitivity analysis due to P_C

The effect of variation of P_C in fuzzy model-I is defined in Table 9. Obviously, increase in P_C has no significant effect in the values of objectives and the total cost.

6. Conclusion

The authors have presented a fuzzy multi-objective inventory model for deteriorating items with shortage. The resulted fuzzy model has been transferred into a non-linear programming and solved using Zimmermann's approach [15]. The proposed model has been solved with fuzzy objective functions in two forms and the results of which has been compared. Results show that increasing the total shortage costs has a decreasing effect on the net profit and finally it remains as a constant rate. Other results show that increasing the upper limit of the net profit in the membership function has an increasing effect on the net profit because the system prefers to make more profit and naturally it results in increasing the satisfaction degree of the deteriorating cost membership function. Since the proposed model is stock depend-

ent demand, the present analysis may be extended with discount, inflation and other related areas.

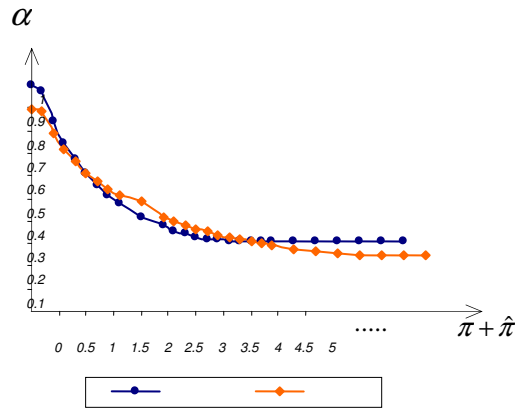


Figure 2. Variations of α due to $\pi + \hat{\pi}$.

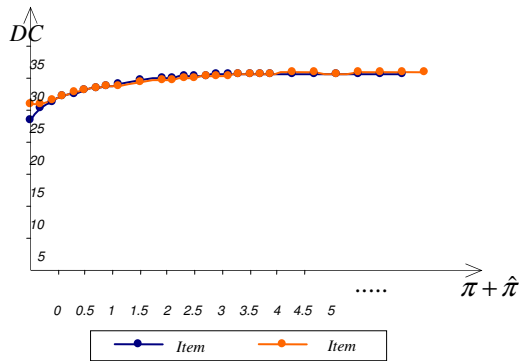


Figure 3. Variations of DC due to $\pi + \hat{\pi}$.

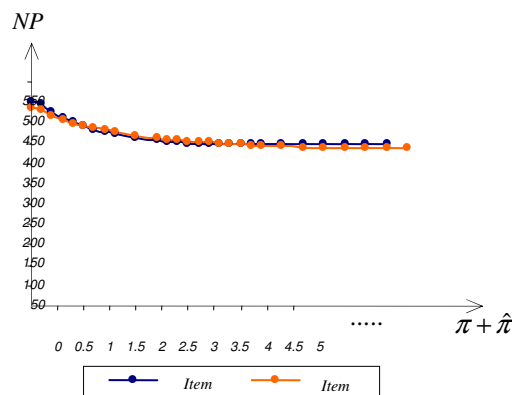


Figure 4. Variations of NP due to $\pi + \hat{\pi}$.

Table 7. Effect of variation in P_{NP} .

| P_{NP} | $NP^*(Q)$ | $DC^*(Q)$ | $TC^*(Q)$ | μ_{NP} | μ_{DC} | μ_C |
|----------|-----------|-----------|-----------|------------|------------|---------|
| 160 | 426.99 | 28.65 | 1933.87 | 0.54 | 0.54 | 0.88 |
| 170 | 426.50 | 28.45 | 1931.56 | 0.57 | 0.57 | 0.89 |
| 180 | 426.05 | 28.28 | 1929.49 | 0.59 | 0.59 | 0.90 |
| 190 | 425.65 | 28.13 | 1927.63 | 0.61 | 0.61 | 0.91 |
| 200 | 425.28 | 27.98 | 1925.92 | 0.62 | 0.62 | 0.91 |
| 220 | 424.63 | 27.74 | 1922.95 | 0.66 | 0.66 | 0.92 |
| 240 | 424.07 | 27.53 | 1920.44 | 0.68 | 0.68 | 0.93 |
| 260 | 423.60 | 27.35 | 1918.60 | 0.70 | 0.70 | 0.94 |
| 280 | 423.18 | 27.19 | 1916.42 | 0.72 | 0.72 | 0.94 |
| 300 | 422.82 | 27.06 | 1914.77 | 0.74 | 0.74 | 0.95 |
| 350 | 422.07 | 26.78 | 1911.47 | 0.77 | 0.77 | 0.96 |
| 400 | 421.50 | 26.57 | 1908.94 | 0.80 | 0.80 | 0.97 |
| 450 | 421.05 | 26.40 | 1906.96 | 0.82 | 0.82 | 0.98 |
| 500 | 420.69 | 26.27 | 1905.34 | 0.84 | 0.84 | 0.98 |

Table 8. Effect of variation in P_{DC} .

| P_{DC} | $NP^*(Q)$ | $DC^*(Q)$ | $TC^*(Q)$ | μ_{NP} | μ_{DC} | μ_C |
|----------|-----------|-----------|-----------|------------|------------|---------|
| 10 | 429.65 | 29.68 | 1946.39 | 0.53 | 0.53 | 0.84 |
| 15 | 434.31 | 31.56 | 1969.10 | 0.56 | 0.56 | 0.76 |
| 20 | 438.29 | 33.22 | 1989.27 | 0.58 | 0.58 | 0.70 |
| 25 | 441.74 | 34.71 | 2007.36 | 0.61 | 0.61 | 0.64 |
| 75 | 445.64 | 38.11 | 2008.71 | 0.64 | 0.82 | 0.64 |
| 100 | 445.64 | 38.11 | 2008.71 | 0.64 | 0.82 | 0.64 |
| 1000 | 445.64 | 38.11 | 2008.71 | 0.64 | 0.82 | 0.64 |
| 10000 | 445.64 | 38.11 | 2008.71 | 0.64 | 0.82 | 0.64 |
| 100000 | 445.64 | 38.11 | 2008.71 | 0.64 | 0.82 | 0.64 |

Table 9. Effect of variation in P_C .

| P_C | $NP^*(Q)$ | $DC^*(Q)$ | $TC^*(Q)$ | μ_{NP} | μ_{DC} | μ_C |
|--------|-----------|-----------|-----------|------------|------------|---------|
| 350 | 427.55 | 28.86 | 1936.45 | 0.51 | 0.51 | 0.89 |
| 450 | 427.55 | 28.86 | 1936.45 | 0.51 | 0.51 | 0.92 |
| 550 | 427.55 | 28.86 | 1936.45 | 0.51 | 0.51 | 0.93 |
| 1000 | 427.55 | 28.86 | 1936.45 | 0.51 | 0.51 | 0.96 |
| 10000 | 427.55 | 28.86 | 1936.45 | 0.51 | 0.51 | 0.99 |
| 100000 | 427.55 | 28.86 | 1936.45 | 0.51 | 0.51 | 0.99 |

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