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Research Note:

Inverse feasible problem

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Abstract

In many infeasible linear programs it is important to construct it to a feasible problem with a minimum parameters changing corresponding to a given nonnegative vector. This paper defines a new inverse problem, called "inverse feasible problem". For a given infeasible polyhedron and an n-vector x^0 a minimum perturbation on the parameters can be applied and then a feasible polyhedron is concluded.

Keywords: Inverse feasible problem; Polyhedron

1. Introduction

In the early few years, inverse optimization problems attracted many operations research specialists and different kinds of inverse problems have been developed by researchers [1]. Many real linear programs may be infeasible. The LP/Infeas contains infeasible linear programming test problems collected by John W. Chinneck, for example, GOSH, GRAN, PANG: these very large, large, and medium size models, respectively, problems arose from British Petroleum operations infeasible linear models [2,3].

This paper introduces a new inverse version problem. It clarifies how a minimum perturbation on the problem parameters of a given infeasible polyhedron could be applied and then a feasible polyhedron corresponding to an n-vector x^0 is concluded. The paper is organized as follows: Section 2 defines the inverse feasible problem (IFP) and then an optimal solution of IFP is proved. Some numerical examples demonstrate the simplicity of applying the procedure in section 3. Conclusion remarks are presented in section 4.

2. Inverse feasibility

Consider the following polyhedron:

$$S(A,b)=\{x:\ Ax=b,\ x\geq 0\}$$

Where A is a $m\times n$ matrix and $b\in\Re^m$. Suppose that $S(A,b)=\phi$. Let x^0 be any nonnegative n-vector.

Definition. The inverse feasible problem is defined as perturb the constraint matrix A to D and the right hand side vector b to d such that x^0 be a feasible solution of $S(D,d)$ and $\|A-D\|_p+\|b-d\|_p$ is minimized, where $\|.\|_p$ is some selected L_p norm. Without loss of generality, this paper considers the L_1 norm. Therefore the inverse feasible problem corresponding to x^0 is as:

$$\begin{aligned}&\min \|A-D\|_1+\|b-d\|_1\\&s.t.\\&\qquad x^0\in S(D,d)\end{aligned}$$

It is equivalent of the following linear programming:

$$\begin{aligned}&\min \sum_{i=1}^{m}\sum_{j=1}^{n}\left(\alpha_{ij}+\beta_{ij}\right)+\sum_{i=1}^{m}\left(\alpha_i+\beta_i\right)\\&s.t.\\&\qquad \sum_{j=1}^{n}\left(\beta_{ij}-\alpha_{ij}\right)x_j^0+\alpha_i-\beta_i=e(i),\ i=1,\ldots,m\\&\qquad \alpha_{ij}\geq 0,\beta_{ij}\geq 0,\alpha_i\geq 0,\beta_i\geq 0, i=1,\ldots,m,\ j=1,\ldots,n\end{aligned}$$

Where $e(i)=b_i-\sum_{j=1}^{n}a_{ij}x_j^0$.

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Assume that $I_1 = \{i : e(i) > 0\}$ and $I_2 = \{i : e(i) < 0\}$. It is obvious that $|I_1 \cup I_2| \geq 1$, because $S(A,b) = \phi$. The following theorem gives an optimal solution of the inverse feasible problem.

Theorem. Let $J = \{j : x_j^0 > 1\}$. An optimal solution arises from the following two cases.

Case 1: $J = \phi$.
In this case

$$\alpha_i^* = \begin{cases} e(i) & \text{if } i \in I_1 \\ 0 & \text{otherwise} \end{cases}$$

$$\beta_i^* = \begin{cases} -e(i) & \text{if } i \in I_2 \\ 0 & \text{otherwise} \end{cases}$$

and $\alpha_{ij}^* = \beta_{ij}^* = 0, i = 1, \ldots, m,\ j = 1, \ldots, n$.

Case2: $J \neq \phi$.
In this case $\alpha_i^* = \beta_i^* = 0, i = 1, \ldots, m$. Let $x_k^0 = \max\{x_j^0 : j \in J\}$ and put

$$\beta_{ij}^* = \begin{cases} \dfrac{e(i)}{x_k^0} & \text{if } i \in I_1 \text{ and } j = k \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_{ij}^* = \begin{cases} \dfrac{-e(i)}{x_k^0} & \text{if } i \in I_2 \text{ and } j = k \\ 0 & \text{otherwise} \end{cases}$$

Proof. Notice that the variables of each constraint in the inverse feasible problem does not appear on any other constrain. This completes the proof.

3. A numerical example

Consider the following simple constraints:

$$\begin{aligned} &2x_1 + x_2 - x_3 + 2x_4 = 1 \\ &3x_1 + 2x_2 + x_3 - 3x_4 = 2 \\ &-2x_1 - 3x_2 + x_3 - 2x_4 = 4 \\ &x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0 \end{aligned}$$

Summation of the first and last equations gives the infeasible equation $x_2 = -2.5$. The inverse feasible problem corresponding to $x^0 = (1,1,1,1)$ (arbitrary) is:

$$\text{Min} \sum_{i=1}^{3}\sum_{j=1}^{4}(\alpha_{ij} + \beta_{ij}) + \sum_{i=1}^{3}(\alpha_i + \beta_i)$$

s.t.

$$\sum_{j=1}^{4}(\beta_{ij} - \alpha_{ij}) + (\alpha_i - \beta_i) = \begin{cases} -3 & i = 1 \\ -1 & i = 2 \\ 10 & i = 3 \end{cases}$$

$$\alpha_{ij} \geq 0, \beta_{ij} \geq 0,$$
$$\alpha_i \geq 0, \beta_i \geq 0,\ i = 1,2,3,\ j = 1,2,3,4$$

According to the theorem an optimal solution is $\beta_1^* = 3, \beta_2^* = 1, \alpha_3^* = 10$ and all other variables are equal to zero. So by changing the right hand side values to $d_1 = 1 - 3 = -2$ $d_2 = 2 - 1 = 1, d_3 = 4 + 10 = 14$, x^0 becomes a feasible solution for $S(D,d) = S(A,d)$.

4. Conclusion

Recent years have seen a great number of researches on the analysis, diagnosis and repair of infeasible linear programming models. In many infeasible linear programs it is important to construct it to a feasible problem with a minimum parameters changing corresponding to a given nonnegative vector. This paper introduces a new inverse version problem, called inverse feasible problem. It considers a minimum perturbation on the parameters of a given infeasible polyhedron and then a feasible polyhedron corresponding to an *n*-vector x^0 is concluded.

References

[1] Ahuja, R. K. and Orlin, J. B., 2001, Inverse optimization. *Operations Research*, 49, 771-783.

[2] Chinneck, J. W. and Dravnieks, E. W., 1991, Locating minimal infeasible constraint sets in linear programs. *ORSA Journal on Computing,* 3(2), 157-168.

[3] Chinneck, J. W., 1992, Viability analysis: a formulation aid for all classes of network models. *Naval Research Logistics*, 39, 531-543.